

Designing Compact Gussets with the Uniform Force Method

LARRY S. MUIR

In 1991 an AISC task group endorsed the uniform force method (UFM) as the preferred method for determining the forces that exist at gusset interfaces. Since that time it has been included in the *AISC Manual of Steel Construction*. The UFM provides a standardized way to obtain economical, statically admissible force distributions for vertical bracing connections. One criticism of the method is that it sometimes results in oddly shaped or disproportionately large gusset plates. To overcome this perceived limitation of the UFM, designers have been seeking out alternate methods.

This paper demonstrates that removing one unnecessary geometrical constraint from the formulation of the UFM will allow greater freedom in gusset geometry, while maintaining the efficiencies that result from the method. A new formulation of the UFM is presented, and the strengths and weaknesses of other proposed design methods are also explored.

THE UNIFORM FORCE METHOD

The uniform force method has been included in the *AISC Manual of Steel Construction* since 1992. The UFM was originally proposed by Thornton (1991) and was based on observations from Richard's (1986) research. In the commonly accepted form, the UFM produces the following force distribution:

$$H_b = \frac{\alpha}{r} P \quad (1a)$$

$$V_b = \frac{e_b}{r} P \quad (1b)$$

$$H_c = \frac{e_c}{r} P \quad (1c)$$

$$V_c = \frac{\beta}{r} P \quad (1d)$$

where

$$r = \sqrt{(\alpha + e_c)^2 + (\beta + e_b)^2} \quad (2)$$

$$\alpha = \tan \theta (\beta + e_b) - e_c \quad (3)$$

In order to satisfy the relationship between α and β , the designer is often forced to use either an oddly shaped or disproportionately large gusset plate. Alternately, moments can be introduced at the connection interfaces. Neither approach is ideal.

ALTERNATIVES TO THE UNIFORM FORCE METHOD

Any viable alternative sought to replace the UFM should meet the following criteria: (1) it must provide a clear procedure to satisfy equilibrium and conform to the basic assumptions made during the analysis and design of the main members (the most important criterion); (2) since the UFM readily accommodates a wide range of geometries and boundary conditions, any alternate method should also be able to accommodate such situations; and (3) it must result in economical designs.

Several alternatives to the UFM have been proposed. Chief among the alternatives are the KISS Method, the parallel force method and the truss analogy method. None of these methods suffer from the constrictive relationship between α and β that exists in the UFM. In other words, these methods can be used with any gusset geometry and do not force the use of oddly shaped or large gusset plates. The strengths and weaknesses of these methods will be explored. In all of the discussions the work-point of the brace is assumed to be located at the intersection of the centerlines of the beam and the column, since this is the typical case.

Larry S. Muir is president of Cives Engineering Corporation and chief engineer of Cives Steel Company, Roswell, GA.

The KISS Method

KISS (Figure 1) is an acronym for “keep it simple stupid,” and the method is simple, as the name implies, and fool-proof, though uneconomical. The method involves delivering the entire horizontal brace component directly to the beam through the beam-to-gusset connection and the entire vertical brace component directly to the column through the column-to-gusset connection. To satisfy equilibrium, moments must be introduced. At the beam-to-gusset the moment is equal to He_b , and at the column-to-gusset the moment is equal to Vc_c .

The KISS Method satisfies two of the three criteria for a viable alternative to the UFM. It satisfies equilibrium and the design and analysis assumptions, and it is universally applicable to all geometries and boundary conditions. However, the presence of the large moments at the connection interfaces makes it an uneconomical choice in practice.

The Parallel Force Method

In the parallel force method, sometimes referred to as the component method (Figure 2), the reactions of the gusset at the beam and column interfaces are assumed to act parallel to the brace force. Since the forces are parallel, they obviously do not intersect at a common point, as is the case with the UFM. Therefore, in order to maintain rotational equilibrium, two choices are available. Either the magnitude of the parallel forces are set so that they balance each other about the work-line of the brace, or moments are added at the beam and/or column interfaces. The additional moments, though lesser in magnitude than the KISS method, adversely impact the economy of the connection.

If the former approach is taken, rotational equilibrium of the beam and column will not generally be satisfied. A moment connection will then have to be added between the beam and the column. Since this will normally be a field-welded connection, this is considered to be an uneconomical alternative in most parts of the country.

In terms of applicability to a variety of boundary conditions, the parallel force method suffers from a major short-coming. Since the forces at both the beam-to-gusset and the column-to-gusset interfaces are assumed parallel to the brace force, a horizontal component will always exist at the column-to-gusset connection. When framing to a column web, this presents a significant design challenge, which will usually be overcome by the addition of column stiffening local to the connection, further reducing the economy of the method.

The parallel force method only satisfies one of the three criteria for a viable alternative to the UFM. It satisfies equilibrium and the design and analysis assumptions, but it is not as economical as the UFM and is not suited to connections made to column webs.

The Truss Analogy Method

The truss analogy method (Figure 3) determines the force distribution on the gusset by modeling the interface forces as a pinned “truss” node located at the center of the brace-to-gusset connection. The truss analogy method suffers the same problem as the parallel force method when attaching to column webs. Additionally, the truss analogy method can result in counterintuitive and uneconomical force distributions. This is illustrated in Figure 3 where the gusset-

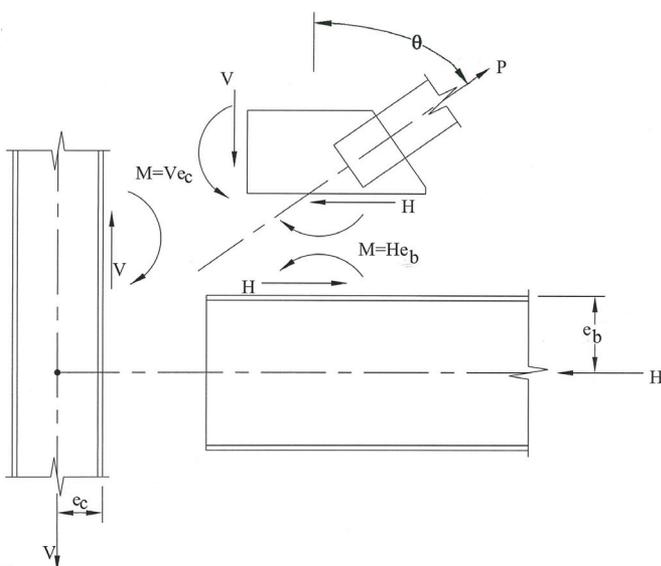


Fig. 1. KISS method.

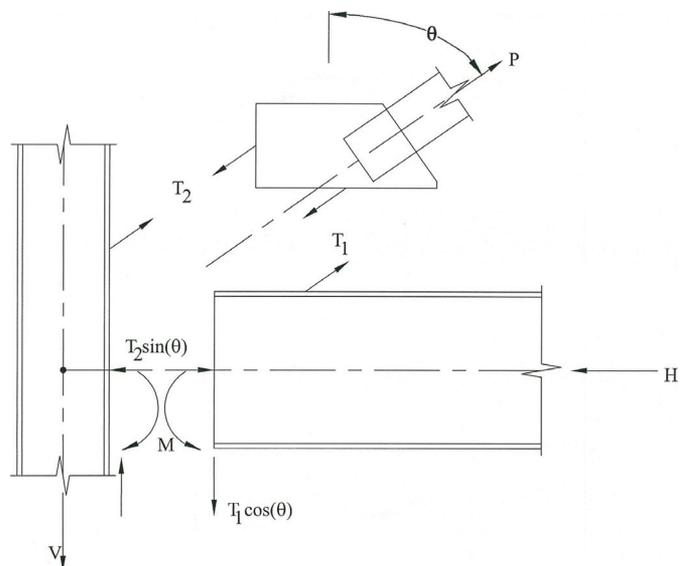


Fig. 2. Parallel force method.

to-column connection delivers only a horizontal component to the column. A formalized treatment of the equilibrium requirements for the beam and column has never been presented and is therefore left to the designer. Often moments are required at all of the connection interfaces in order to satisfy equilibrium.

The truss analogy method satisfies none of the criteria for a viable alternative to the UFM.

A GENERALIZED UFM

Since none of the alternatives investigated appear to provide better results than the UFM, it is advantageous to make adjustments to the formulation of the UFM to make it more applicable to compact gussets.

The goal of the UFM was to derive a procedure to obtain statically admissible force distributions, which would produce no moments at the connection interfaces and would be applicable to a wide range of geometries and boundary conditions. However, the procedure includes an additional constraint that unnecessarily limits its applicability. The force at the gusset-to-column interface, $\sqrt{V_c^2 + H_c^2}$, is forced to pass through a point that lies a distance, e_b , above the work-point.

Since there is a perceived problem with the UFM that can be overcome by removing this constraint, it is advantageous to eliminate it from the method. In order to do so, the problem must first be defined. There are essentially three elements involved: the beam, the column, and the gusset. The brace is neglected since it is assumed to carry only axial force and is not part of the indeterminate system. Each of the three members is subjected to three forces. In order for

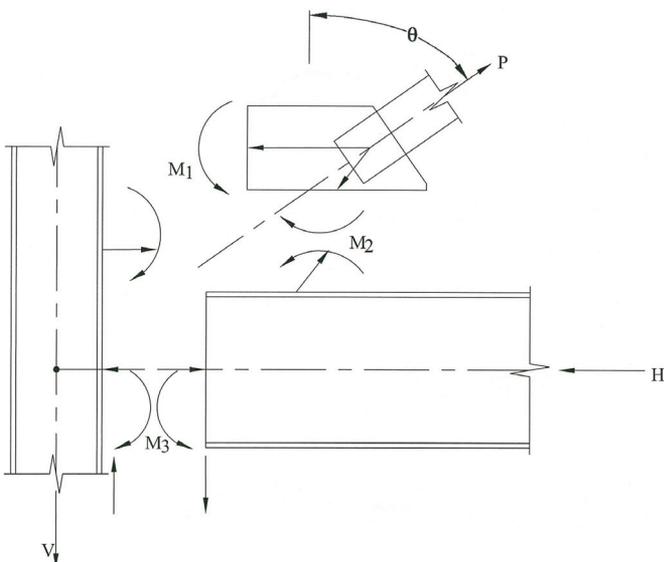


Fig. 3. Truss analogy method.

moments to be eliminated from the interfaces the forces applied to each element must intersect at a single point. These points of intersection are referred to as control points.

The Beam

It is easiest to begin with the beam (Figure 4), since the location of its control point is evident. The three forces applied to the beam are the horizontal component of the brace, H , the beam-to-gusset force, $\sqrt{V_b^2 + H_b^2}$, and the beam-to-column force, $\sqrt{V_c^2 + H_c^2}$. The horizontal component of the brace is resisted along the centerline of the beam and intersects the beam-to-column force at the point $(e_c, 0)$. Therefore, the beam-to-gusset force must also pass through this point. From this we find that

$$\frac{V_b}{H_b} = \frac{e_b}{\alpha} \quad (4)$$

The Gusset

The three forces applied to the gusset (Figure 5) are the brace force, P , the beam-to-gusset force, $\sqrt{V_b^2 + H_b^2}$, and the gusset-to-column force, $\sqrt{V_c^2 + H_c^2}$. In order to eliminate moments at the interfaces, these three forces must intersect at a single point. Since the slope of the brace force, $1/\tan(\theta)$, and the slope of the beam-to-gusset force, e_b/α , are known, the intersection can be determined. The gusset control point is:

$$\left(\frac{e_b e_c \tan(\theta)}{e_b \tan(\theta) - \alpha}, \frac{e_b e_c}{e_b \tan(\theta) - \alpha} \right)$$

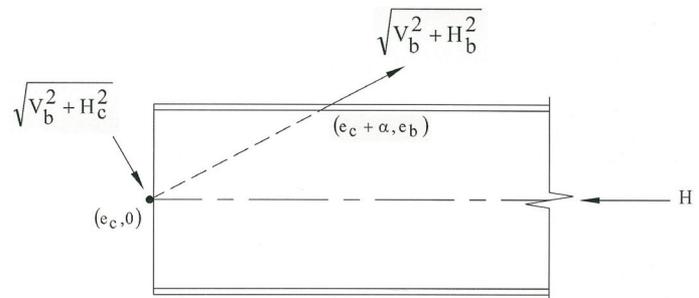


Fig. 4. Beam free body diagram.

With the geometry and force distribution established, a new form of the UFM has been derived without the somewhat arbitrary constraint on the location of the column control point. Without this constraint, α and β can be set to any convenient values. This removes the need to consider the moments caused by $\bar{\alpha}$ and $\bar{\beta}$, where $\bar{\alpha}$ is the actual distance from the face of the column flange to the centroid of the gusset-to-beam connection, and $\bar{\beta}$ is the actual distance from the face of the beam flange to the centroid of the gusset-to-column connection.

However, there may still be a need to redistribute the vertical reaction delivered to the beam, V_b . This counteracting force is referred to as ΔV_b . ΔV_b can be introduced into this new formulation easily to produce the full spectrum of force distributions that can exist in the connection while maintaining column-to-gusset and beam-to-column connections free of moments. It is assumed that moments at the column-to-gusset and beam-to-column connections are uneconomical and therefore undesirable.

Of course the introduction of ΔV_b disrupts the established equilibrium and adjustments must be made. The adjustment involves introducing a moment at the beam-to-gusset interface. This moment can be calculated as:

$$M_b = H_b e_b - (V_b - \Delta V_b) \alpha \quad (14)$$

Column Moment

A moment gradient will exist in the column whether using the original formulation or the new formulation of the UFM presented in this paper. Using the original formulation, the moment will be zero at the intersection of the top of steel elevation and the centerline of the column. In the new formulation, the moment may be either positive or negative throughout the section of the column bounded by the connection or the moment may be zero at some section similar to the original formulation. In either case the maximum moment the column will be subjected to can be determined as:

$$M_c = \max \left\{ V_c e_c, (V_c e_c - H_c (e_b + \beta)) \right\} \quad (15)$$

Since the choice of column section will usually be governed by buckling and the column is restrained from buckling local to the brace connection, it is normal practice to neglect this moment. For this reason, the moment internal to the column is not mentioned in the AISC *Steel Construction Manual* (AISC, 2005) discussion of the UFM.

An Example

The forces on the connection shown in Figure 7 will be calculated to demonstrate the new formulation.

$$\alpha = \frac{27.75}{2} + 0.5 = 14.375 \text{ in.}$$

$$\beta = \frac{13}{2} = 6.5 \text{ in.}$$

$$H_c = \frac{\cos(55^\circ)(7)}{(12 + 6.5)} 100 = 21.7 \text{ kips}$$

$$V_b = \left[\frac{(12)(\sin(55^\circ)(12 + 6.5) - \cos(55^\circ)(7))}{(14.375)(12 + 6.5)} \right] (100) \\ = 50.3 \text{ kips}$$

$$H_b = 100 \sin(55^\circ) - 21.7 = 60.2 \text{ kips}$$

$$V_c = 100 \cos(55^\circ) - 50.3 = 7.06 \text{ kips}$$

Summing moments on the beam about the beam control point produces:

$$V_b \alpha - H_b e_b = 50.3(14.375) - 60.2(12) \approx 0 \text{ kip-in.}$$

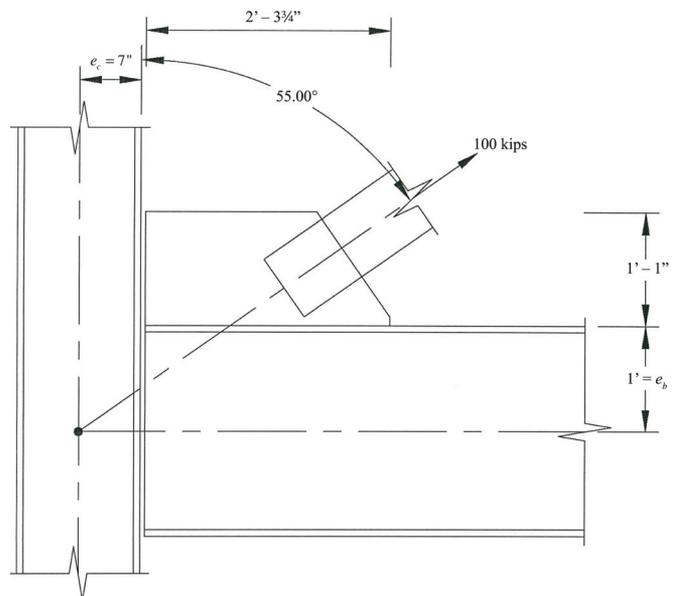


Fig. 7. Example.

Summing moments on the gusset about the work-point produces:

$$V_b(\alpha + e_c) - H_b e_b + V_c e_c - H_c(e_b + \beta) = 50.3(14.375 + 7) - 60.2(12) + 7.06(7) - 21.7(12 + 6.5) \approx 0 \text{ kip-in.}$$

Summing moments on the column about the beam-to-column connection produces:

$$P \cos(\theta) e_c - H_c(e_b + \beta) = 100 \cos(55^\circ)(7) - 21.7(12 + 6.5) \approx 0 \text{ kip-in.}$$

Note that:

$$\tan(\theta)(\beta + e_b) - e_c = \tan(55^\circ)(6.5 + 12) - 7 = 19.4 \neq \alpha$$

For completeness the vertical coordinate of the column control point can be calculated as:

$$y_{ccp} = e_b \left(\frac{\tan(\theta)(e_b + \beta) - e_c}{\alpha} \right) = (12) \left(\frac{\tan(55^\circ)(12 + 6.5) - 7}{14.375} \right) = 16.2 \text{ in.}$$

It may be noted that for this case the term V_b is significantly larger than would be obtained using the traditional UFM. As is the case with the traditional UFM, a ΔV_b can be introduced to manipulate the distribution of vertical force. Taking ΔV_b equal to 13.1 kips produces the same distribution of vertical force that is obtained from the UFM when all parameters except α are held constant.

As can be seen from Table 1, which presents a comparison of the traditional UFM to the modified UFM, each can be modified to produce identical results. This is to be expected since each must satisfy equilibrium. The primary advantage to the new formulation is that it eliminates the need for the modifiers $\bar{\alpha}$ and $\bar{\beta}$. Also the new formulation makes it easier to overcome the perceived limitations of the UFM.

Parameters	Traditional UFM		Modified UFM	
	without $\bar{\alpha}$	with $\bar{\alpha}$	without ΔV_b	with ΔV_b
α	19.4	19.4	14.4	14.4
$\bar{\alpha}$	-	14.4	-	-
β	6.5	6.5	6.5	6.5
V_b	37.2	37.2	50.3	37.2
H_b	60.2	60.2	60.2	60.2
V_c	20.2	20.2	7.06	20.2
H_c	21.7	21.7	21.7	21.7
ΔV_b	-	-	-	13.1
M_b	-	188	-	188

OTHER PRACTICES THAT CAN REDUCE THE GUSSET PROFILE

Having eliminated the geometrical constraints on gusset size from the UFM, attention can be turned to other steps that can be taken to reduce the gusset profile.

The Whitmore Section

The Whitmore section is commonly accepted to be an area, which extends at a 30° angle from the edges of the brace-to-gusset connection along the length of the connection. The area beyond this section is assumed to be ineffective in terms of gross tension yielding and compression buckling of the gusset. It is common practice to try to include all of the allowed Whitmore section within the gusset, but it is not a requirement to do so. By allowing the edges of the gusset plate to encroach on the Whitmore section, the profile of the gusset can be reduced.

Weld Size

It is common practice to attempt to limit fillet weld sizes to those that can be applied in a single pass, usually $\frac{5}{16}$ in. This greatly enhances connection economy, since the number of passes required to complete a weld increases disproportionately with the leg size. To maintain a single pass weld, the gusset plate dimensions, particularly at the beam-to-gusset connection, are often increased. The gusset profile can be reduced by allowing multiple pass welds to be used, but only with increased fabrication costs.

Bolt Type

If reducing the gusset profile is of paramount concern, the strongest possible bolt configuration should be employed. Slip-critical connections should be avoided since they will require more bolts and therefore a larger gusset profile. Likewise, if the threads will be excluded from the shear plane, which is usually the case for heavily loaded bracing connections, then the “X-value” for the bolts should be used. Providing a detail that places the bolts in double shear at the brace-to-gusset connection also helps to reduce the gusset profile.

CONCLUSIONS

The UFM, as currently presented in the Manual, contains an unnecessary constraint on the location of the column control point. This constraint often gives designers the perception that the method is ill suited to the design of compact gusset plates.

By eliminating the unnecessary constraint in the new formulation, force distributions can be derived that consist of only shear and axial forces at the connection interfaces. The new formulation also simplifies the UFM by eliminating the need for $\bar{\alpha}$ and $\bar{\beta}$.

By manipulating the term ΔV_b , designers can obtain the full spectrum of force distributions that can exist in the connection while maintaining column-to-gusset and beam-to-column connections free of moments.

NOTATION

e_b	=	one-half the depth of the beam
e_c	=	one-half the depth of the column
y_{ccp}	=	vertical coordinate of the column control point
P	=	brace load
H	=	horizontal component of the brace load

H_b	=	shear force on the beam-to-gusset connection
H_c	=	axial force on the beam-to-column and gusset-to-column connections (assumes no transfer force)
M_b	=	moment on the beam-to-gusset connection
V	=	vertical component of the brace load
V_b	=	shear force on the beam-to-column connection and axial force on the beam-to-gusset connection
V_c	=	shear force on the gusset-to-column connection
ΔV_b	=	change in the distribution of vertical load
α	=	distance from the face of the column flange or web to the centroid of the gusset-to-beam connection
β	=	distance from the face of the beam flange to the centroid of the gusset-to-column connection
$\bar{\alpha}$	=	actual distance from the face of the column flange to the centroid of the gusset-to-beam connection (This term is not required in the new formulation.)
$\bar{\beta}$	=	actual distance from the face of the beam flange to the centroid of the gusset-to-column connection (This term is not required in the new formulation.)

REFERENCES

- AISC (2005), *Steel Construction Manual*, 13th Edition, American Institute of Steel Construction, Inc., Chicago, IL.
- Richard, R.M. (1986), “Analysis of Large Bracing Connection Designs for Heavy Construction,” *National Steel Construction Conference Proceedings*, American Institute of Steel Construction, Inc., Chicago, IL.
- Thornton, W.A. (1991), “On the Analysis and Design of Bracing Connections,” *National Steel Construction Conference Proceedings*, American Institute of Steel Construction, Inc., Chicago, IL.

CLOSURE

Designing Compact Gussets with the Uniform Force Method

Paper by Larry S. Muir
(First Quarter, 2008)

Closure by Larry S. Muir

I would like to begin by thanking Mr. Arias for his interest in and comments regarding my paper. I believe that an open and vigorous discourse is the best way to advance our understanding and practice of engineering.

Mr. Arias addresses three separate issues, which I will try to restate:

1. I have found only one of a number of possible solutions to the problem.
2. I have applied arbitrary geometric constraints to the analysis, and my analysis does not reflect the behavior of the connection.
3. I present an alternative that uses ΔV_b to arbitrarily manipulate the distribution of vertical forces in the connection.

I agree with all three of Mr. Arias' points enumerated here. However, I disagree with the conclusions developed from these points. It is my understanding that Mr. Arias's main problem with the approach presented in my paper is that it is arbitrary and does not accurately reflect the true behavior of the connection. From this he concludes that the procedure may result in inadequate designs and that the traditional UFM more accurately reflects the behavior of the connection and therefore results in safer designs.

Larry S. Muir is the president of Cives Engineering Corporation and chief engineer at Cives Steel Company, Roswell, GA.

I contend that no one—not Mr. Arias, not myself, not Dr. Thornton, the originator of the UFM—can accurately predict the behavior of any connection. That is why all connection design—and, in all likelihood, virtually all structural steel design—is accomplished based, either implicitly or explicitly, on the Lower Bound Theorem. The Lower Bound Theorem states that the applied external forces in equilibrium with the internal force field are less than or, at most, equal to the applied external force that would cause failure, provided that all the limit states are satisfied and sufficient ductility exists to allow redistribution of the forces. In other words, as long as sufficient ductility is present and all applicable limit states are satisfied, design can safely proceed based on any arbitrary distribution of forces, as long as the distribution satisfies equilibrium. If this was not true, designs would quickly grind to a halt as we constructed and calibrated, through physical testing, highly complex finite element models for every detail and possible load case for our designs.

Mr. Arias brings up many arguments that are certainly true. There will undoubtedly be some moment present in the physical connection at the beam-to-column interface. However, this moment will be limited to some value less than the ultimate strength of the beam-to-column connection. As the loads imposed on the connection approach the connection strength, the elements will begin to yield and therefore shed load to stiffer elements. As it turns out, neglecting the rotational stiffness of this connection and the resulting imposed moments in the analysis actually adds to, and not subtracts from, the safety of the connection. Any additional restraint will serve to strengthen, not weaken, the structure.

As Mr. Arias states, increasing the β dimension of the connection will tend to make it more rigid at the gusset-to-column interface. This will, as Mr. Arias asserts, draw moment from the gusset-to-beam interface. The prediction that no moment exists at the gusset-to-column interface is most certainly incorrect, as are all the other forces predicted by the proposed procedure. Some of the predicted forces are too

high, some are too low, but still the resulting design is safe and will carry the loads, or else the Lower Bound Theorem is wrong and so too are countless structures in service.

This same logic justifies the use of ΔV_b to manipulate the distribution of the forces in the connection. The use of ΔV_b predates my paper and has been present in the AISC *Manual* for many years. It is used primarily where the beam end connection is subjected to a high shear load due to gravity loads, so that it cannot resist the additional load imposed by the bracing with a typical connection. In some instances, the additional shear induced by the bracing may be such that the beam web itself is overstressed when subjected to the forces predicted by the UFM. If the beam and its connections maintained their stiffness throughout loading and then suddenly snapped like glass, it would be inappropriate to apply ΔV_b —but this is not how steel behaves.

Finally, Mr. Arias suggests that the traditional UFM is inherently superior to the procedure presented in the paper. Based on his previous arguments regarding the generalized UFM presented in the paper, this implies he feels the

traditional UFM is less arbitrary than the generalized method. In fact, it could be argued that the traditional UFM is actually more arbitrary in the constraints it chooses to impose on the force distribution. When he derived the traditional UFM, Dr. William Thornton arbitrarily chose to pass the forces V_c and H_c through a point at the intersection of the top of steel and the face of the column. This ensured that no moment would exist in a section cut through the column at the top of steel. This choice was based in part on figures shown in Blodgett's *Design of Welded Structures*. It resulted in more elegant-appearing equations for the interface forces than my proposed generalized method, but actually contained one additional arbitrary geometric constraint than the generalized procedure.

In conclusion, the procedure presented in my paper was never intended to accurately predict the forces present in the connection. It was intended instead as an improvement to an existing tool by which an admissible force distribution can be obtained that has been proven through use to produce safe and economical designs.

DISCUSSION

Designing Compact Gussets with the Uniform Force Method

Paper by Larry S. Muir
(First Quarter, 2008)

Discussion by Ramon F. Arias

The writer submits this discussion with some trepidation as he recognizes that the author of the paper is a well-known authority in the design of connections, and who has made valuable contributions to the AISC codes and to books on the subject.

Prior to discussing the substance of the author's paper, a typo should be corrected as follows:

The author's equation (8) reading $\Sigma F_x = 0 = H_c - H_b$ should read $\Sigma F_x = 0 = P \sin \theta - (H_b + H_c)$. This correction does not affect the author's analysis.

The first point of discussion is the author's approach. It appears that the author's solution to the compact gusset corresponds to one of two boundary conditions that would frame the actual solution.

In discussing this, two geometric points should be noted: point B_o on the beam axis at the column flange and point C_o on the column axis at the level of the top of the beam. In the traditional Uniform Force Method (UFM), the force (H_b, V_b) passes through point B_o and the force (H_c, V_c) passes through point C_o . In other words, the moment M_{bo} of the forces H_b and V_b relative to point B_o and the moment M_{co} of the forces H_c and V_c relative to point C_o are both zero.

In the author's solution to the compact gusset, the force (H_b, V_b) passes through B_o but the force (H_c, V_c) is left to drift away from C_o and all of this in such a way that the equations of equilibrium are satisfied. In other words, $M_{bo} = 0$ and M_{co} differs from zero. Under these conditions, expressions are derived for H_c , V_b , H_b and V_c as exhibited by the author's equations (10), (11), (12) and (13), respectively.

Additionally, the moments M_{bo} and M_{co} defined earlier are:

$$M_{bo} = 0 = H_b e_b - V_b \alpha \quad (13.1)$$

$$M_{co} = H_c \beta - V_c e_c \quad (13.2)$$

Equations 13.1 and 13.2 are not in the author's paper, but they can be easily derived from his analysis.

The other boundary condition consists of letting the force (H_b, V_b) drift away from point B_o while keeping the force (H_c, V_c) through C_o and satisfying the equations of equilibrium. That is to say, $M_{co} = 0$ while M_{bo} differs from zero. Under these conditions the analysis gives the following results:

$$H_c = e_c P \{ \cos \theta - e_b \sin \theta / (e_c + \alpha) \} / \beta \quad (10a)$$

$$V_b = e_b P \sin \theta / (e_c + \alpha) \quad (11a)$$

$$H_b = P \sin \theta - H_c \quad (\text{same as the author's equation 12})$$

$$V_c = P \cos \theta - V_b \quad (\text{same as the author's equation 13})$$

$$M_{bo} = H_b e_b - V_b \alpha \quad (\text{same as 13.1 above})$$

$$M_{co} = 0 = H_c \beta - V_c e_c \quad (\text{same as 13.2 above})$$

If the author's approach is correct, the solution of the problem created by the compact gusset should be located between these boundary conditions. In this solution, both M_{bo} and M_{co} are different from zero. These moments are relatively small. M_{co} is usually neglected and M_{bo} can be easily accommodated in the beam to column connection.

Table 1a shows a comparison between the two boundaries and includes a solution consisting on a weighted sum of the values from the boundary conditions. The weight factor for the boundary 1 values is $k_1 = d_c / (d_b + d_c)$ and for boundary 2 is $k_2 = d_b / (d_b + d_c)$, where d_b and d_c are the distances from points B (centroid of the beam-to-column connection)

Ramon F. Arias is a principal civil engineer with Babcock Hitachi K.K., Kure Division, Vancouver, BC.

Table 1a. Comparison between Muir's solution (Boundary 1), Boundary 2, and a Weighted Solution			
Parameters	Boundary 1	Boundary 2	Weighted Solution
V_b	50.3	46.0	47.9
H_b	60.2	69.7	65.4
V_c	7.09	11.4	9.46
H_c	21.7	12.2	16.5
M_{bo}	0.0	175.0	96.8
M_{co}	91.4	0.0	40.8

and C (centroid of the column-to-gusset connection) to the working point (WP) or point of intersection of the beam and column axes. Then,

$$V_b = k_1 V_{b1} + k_2 V_{b2}$$

$$H_b = k_1 H_{b1} + k_2 H_{b2}$$

and so on, where V_{b1} , H_{b1} , V_{b2} and H_{b2} indicate the reaction values corresponding to boundaries 1 and 2.

Of course, the two boundary conditions and the weighted solution satisfy $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M = 0$.

A second point of discussion is the approach adopted by the author and continued in this discussion, in which geometric constraints are imposed to determine the connection forces.

The solution to the compact gusset cannot be any of the two boundary conditions or the weighted solution because they contain geometric restrictions that are imposed by the designer (the forces must pass through certain points and must be oriented in certain directions). However, the behavior of the connection under the axial force does not have to follow arbitrary geometric constraints.

Since the forces are concurrent and their sum is zero, it follows as a result that the sum of their moments relative to any point is also zero. This approach is arbitrary as it gives solutions without having to make implicit or explicit reference to the moment caused by the local eccentricity of the brace axial force relative to the centroid G of the connections of the gusset to beam and column. The fact that the sum of moments is zero should be a condition of the problem, not a result of imposed geometric constraints. In other words, the analysis must reflect the behavior of the connection, and the geometric constraints must be relaxed accordingly.

A third and final point of discussion is the adoption by the author of a force, ΔV_b , to manipulate force, V_b , resulting in a moment, M_b , assigned to the beam to gusset connection and given by the author's equation (14). Actually, what is adopted is a couple, $(\Delta V_b, \alpha)$, with forces, ΔV_b , oriented vertically so that they only affect V_b and V_c , acting at points B and C .

The results of this manipulation are shown in Table 1 of the author's paper (refer to the column entitled "Modified UFM with ΔV_b " in this table). The adoption of a couple of forces that act in the vertical direction and applied at points that do not fall in a horizontal line is arbitrary. The forces should be oriented perpendicular to line BC and it should affect all connection forces.

Additionally, the author does not explain why the moment, M_b , required to balance the couple, $(\Delta V_b, \alpha)$, is applied to the beam-to-gusset interface. Presumably, the reason is that this connection is more rigid than the column-to-gusset connection. However, it could be argued that the design of a more compact gusset with a larger β , as in the author's example, results in an increased rigidity of the column-to-gusset connection compared to the concentric gusset connection case. Based on this increase, the moment should be assigned to the column-to-gusset connection. Furthermore, assigning the moment M_b to the beam-to-gusset connection interface appears to defeat the purpose of the author's solution as this moment must also be applied to the beam to column connection.

The writer feels that if there is a need to manipulate the results, an alternative is to introduce a moment, M_{bc} , affecting all four connection forces. The moment, M_{bc} , should consist of a couple, $(\Delta F, d_{bc})$, where d_{bc} is the distance between B and C ($d_{bc}^2 = \alpha^2 + \beta^2$). The force, ΔF , would be selected so as to result in the desired values of the connection forces. Then, a balancing moment, $-\Delta M$, would have to be assigned to one of the two connections.

In summary, the author has identified a boundary condition to the solution of the compact gusset. However, this boundary condition has been reached by adopting geometric constraints that appear to invalidate it. This also applies to the second boundary condition and the weighted solution presented here. Some of the resulting connection forces in the boundary conditions, including the author's solution, could be underestimated. The introduction by the author of a force, ΔV_b , to manipulate the results of his analysis also seems arbitrary.

A solution to the compact gusset design should (1) be based on the UFM, (2) with the equations of equilibrium as condition of its solution, (3) include in the analysis the local eccentricity of the brace force relative to the centroid G , (4) be reduced to the traditional UFM solution under the right circumstances, (5) not include arbitrary geometric constraints, and (6) be easily expanded to accept small eccentricities created by the brace axial force not passing through WP. This solution would have M_{bo} and M_{co} different from zero, except for the special case where the centroid G is located on the axis of the brace.

NOTATION

d_b	= distance from the beam-to-gusset connection to the working point	H_b, H_{b1}, H_{b2}	= shear force on the beam-to-gusset connection
d_c	= distance from the column-to-gusset connection to the working point	H_c, H_{c1}, H_{c2}	= tension force on the column-to-gusset connection
d_{bc}	= distance between the beam-to-gusset and column-to gusset connection centroids	M_b	= moment on the beam-to-gusset connection
e_b	= one-half the depth of the beam	M_{bc}	= moment introduced to manipulate the reactions
e_c	= one-half the depth of the column	M_{bo}	= moment on point B_o
k_1, k_2	= weight factors	M_{co}	= moment on point C_o
B	= centroid of the beam-to-gusset connection	V_b, V_{b1}, V_{b2}	= tension force on the beam-to-gusset connection
B_o	= centroid of the beam-to-column connection	V_c, V_{c1}, V_{c2}	= shear force on the column-to-gusset connection
C	= centroid of the column-to-gusset connection	WP	= point of intersection of the beam and column axes
C_o	= point on the column axis at the level of the top of the beam	α	= distance from face of column to centroid of beam-to-gusset connection
G	= centroid of the combined beam-to-gusset and column-to-gusset connections	β	= distance from face of beam flange to centroid of column-to-gusset connection
		ΔF	= force adopted to manipulate all reactions ($\Delta F = M_{bc}/d_{bc}$)
		ΔV_b	= change in the distribution of vertical reactions
		(X, Y)	= force resulting from the vector addition of forces X and Y
		(X, y)	= pair for forces X acting in opposite directions and at a distance

