

# **AN ALTERNATIVE APPROACH TO DESIGN OF ECCENTRICALLY LOADED BOLT GROUPS**

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## **ABSTRACT**

The instantaneous center method used to calculate the ultimate capacity of eccentrically loaded bolt groups assumes that the bolts are the controlling element in the connection in order to obtain the maximum permissible load based on bolt shear alone. In other words, a weak bolt, strong plate model is assumed. Prior to the AISC LRFD 3<sup>rd</sup> Edition Manual (AISC 2001), this approach was satisfactory since the designer did not have to be concerned that the bolts would tear out through the material as long as an edge distance of 1.5 times the bolt diameter was maintained. The increased likelihood that bolt tear-out will be the limiting failure mode under the 3<sup>rd</sup> Edition Specification, requires that a new model be developed to better approximate the true maximum permissible load for the connection. Using the Lower Bound Theorem of Limit Analysis, the authors have developed an iterative weak plate, strong bolt model which maximizes connection capacity for cases where bolt tear-out, and not bolt shear, is the limiting factor.

## **INTRODUCTION**

The usual approach to eccentrically loaded bolt groups assumes that the bolts have less strength than the plates or members they connect. This is called the “weak bolt/strong plate” model.

When thin plates or members are used, and bolts are close to the edges of plates or members, the bearing strength of the plate or member at these bolts can be less than the shear strength of the bolts.

In this case, the “weak bolt/strong plate” model is not correct, because the force distribution based on bolt shear strength, assumed with this model cannot be achieved.

This paper presents a “weak plate/strong bolt” model which can be used to supplement the “weak bolt/strong plate” model. The capacity of the connection will be the greater of the values that result from the two models, but will not exceed the capacity obtained using the “weak bolt/strong plate” considering only bolt shear.

## **THE WEAK BOLT/STRONG PLATE MODEL**

This is the model that assumes that an eccentrically-loaded bolt group rotates about an “instantaneous center of rotation” (AISC, 2001). This instantaneous center (ic) determines the forces and direction of these forces on each bolt. The method can use an elastic or an inelastic constitutive equation, and the compatibility equation usually assumes that bolt deformation is linearly proportional to distance from the ic. This method is well established,

and computer programs and design charts are available to simplify its use.

However, this model does not allow for the situation that exists when bolts close to an edge cannot develop the force dictated by the ic location. This is a strong bolt/weak plate situation where the bolt force is dictated by the bearing strength of the plate, rather than the shear strength of the bolt.

If the force induced in a bolt close to an edge exceeds the edge distance bearing strength, all bolts in the group must have their forces reduced in the ratio of the edge distance strength to the bolt shear strength. This must be done to guarantee that the calculated location of the ic does not change. If the above ratio is 50%, the capacity of the connection is reduced to 50% of what the weak bolt/strong plate model would otherwise predict. It can be seen that the strength of one bolt in a group can degrade the strength of the entire group. The weak plate/strong bolt model is introduced to mitigate this strength degradation.

### THE WEAK PLATE/STRONG BOLT MODEL

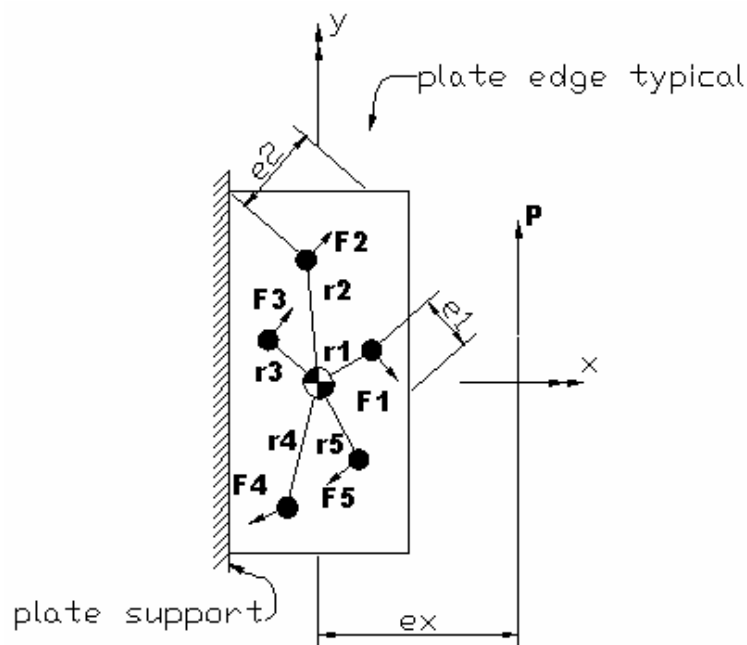


Figure 1. Typical bolt group and plate – weak plate/strong bolt model.

Consider Fig. 1. This shows a typical plate and bolt group. In the following, bold face symbols are used to represent vectors. Point O is any arbitrary point used as an origin. The centroid of the bolt group will usually be used.  $F_i$  is the force on the  $i$ -th bolt,  $r_i$  is the position of the  $i$ -th bolt with respect to the origin, and  $e_i$  is the edge distance for the  $i$ -th bolt, measured from the edge of the bolt hole to the edge of the plate, along as the line of action of the force  $F_i$ .  $P$  is the applied load,  $e_x$  is the eccentricity with respect to the origin, and  $i$  and  $j$  are unit vectors in the  $x$  and  $y$  directions, respectively.

Note that in Fig. 1, the bolt forces  $F_i$  are not necessarily perpendicular to the location radii  $r_i$ . As noted above, the point O, used as an origin for the  $r_i$  vectors, is completely arbitrary, but the bolt group centroid is a convenient point to use.

There is no ic involved in this method. Instead, the bolt forces are each allowed to achieve any magnitude and direction that will maximize the design strength of the connection, subject

to the plate bearing, bolt shear, and equilibrium constraints.

In the notation of Fig. 1, the problem of the weak plate/strong bolt can be formulated as:

Find  $F_i$  such that  $P = \sum_{i=1}^n F_i \bullet \mathbf{j} \rightarrow \max$  subject to the constraints:

$$\sum_{i=1}^n F_i \bullet \mathbf{i} = 0$$

$$\sum_{i=1}^n \mathbf{V}_i \times F_i - e_x \mathbf{i} \times P \mathbf{j} = 0$$

$$F_i \bullet F_i \leq (\phi 1.2 F_u t)^2 e_i^2 \quad i = 1, 2, \dots, n$$

$$1.2 F_u t e_i \leq 2.4 F_u d t \quad i = 1, 2, \dots, n$$

$$1.5 F_u t e_i \leq 2.4 F_u d t \quad i = 1, 2, \dots, n$$

$$F_i \bullet F_i \leq \phi R_{bs}^2 \quad i = 1, 2, \dots, n$$

In the above,  $t$  is the plate thickness,  $F_u$  is the plate strength,  $d$  is the bolt diameter,  $e_i^2 = \mathbf{e}_i \bullet \mathbf{e}_i$ , and  $n$  is the number of bolts. The bolt shear constraint is added to insure that bolt shear limit state  $R_{bs}$  is not exceeded.

Because of the Lower Bound Theorem of Limit Analysis, any solution to the above problem will be less than the collapse solution. The problem is formulated as a non-linear programming problem. The solution, which is the greatest lower bound, approximates the actual collapse solution and will be less than or at most equal to the collapse solution.

Let the solution to the above problem, the weak plate/strong bolt problem, be denoted by  $P_p$ . If the conventional weak bolt/strong plate problem solution is denoted by  $P_b$ , then the proposed capacity is  $P = \max\{P_p, P_b\}$ .

## SOLUTIONS TO SOME SPECIFIC EXAMPLES

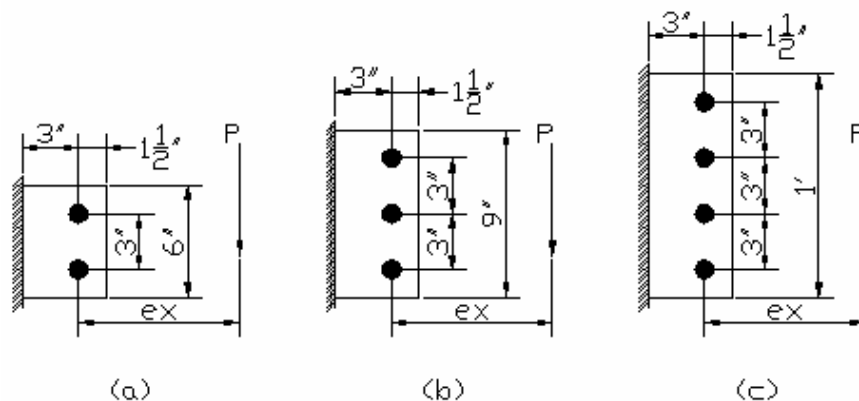


Figure 2. Specific solved cases.

To date, the authors have developed ad hoc solutions to the above general problem for the examples shown in Fig. 2.

The simplest of these cases, the two-bolt group subjected to a vertical eccentric load, Case (a) in Fig. 2, provides a good example of the procedures which can be used to optimise the connection. First the equations limiting the connection capacity are derived which are independent of the direction of the forces. These limit states are bolt shear and bearing, and are defined by the following equations, where  $\eta$  represents the percentage of the vertical force resisted by the first bolt,  $s$  is the bolt spacing, and  $e_x$  is the eccentricity:

$$\text{Capacity}_{bs1} = \frac{\min(\phi R_{bs}, 2.4F_u dt)}{\sqrt{\eta^2 + \left(\frac{e_x}{s}\right)^2}} \quad \text{and} \quad \text{Capacity}_{bs2} = \frac{\min(\phi R_{bs}, 2.4F_u dt)}{\sqrt{(1-\eta)^2 + \left(\frac{e_x}{s}\right)^2}}$$

Since the equation for the bolt shear limit state represents the strong-plate/weak-bolt model, and the bolt capacity is optimised when the bolts vertical load is distributed evenly between the bolts ( $\eta = 0.5$ ) its validity can be proven by re-writing it as a solution for the C-value given in the Table 7-17 (AISC, 2001) as follows:

$$C = \frac{1}{\sqrt{0.25 + \left(\frac{e}{s}\right)^2}}$$

It should be noted that the values obtained using this equation are slightly higher than those shown in Table 7-17 because of the empirical nature of the load deformation equation used by AISC, which results in a maximum normalized force of 0.982 instead of 1.

Next equations for the connection capacity based on bolt tear-out are derived. Under downward vertical loading it is clear that the top bolt may tear-out through the top of the plate, while the bottom bolt may tear-out through the side of the plate. This results in the following capacities:

$$\text{Capacity}_{to-top} = \phi(1.5)(F_u)(t) \left[ \frac{L_{cv}}{\eta} - \frac{\phi_h}{2\sqrt{\left(\frac{e}{s}\right)^2 + \eta^2}} \right]$$

$$\text{Capacity}_{to-side} = \phi(1.5)(F_u)(t) \left[ \frac{L_{ch}}{1-\eta} - \frac{\phi_h}{2\sqrt{\left(\frac{e}{s}\right)^2 + (1-\eta)^2}} \right]$$

In the above  $\phi_h$  is the hole diameter and  $L_{cv}$ ,  $L_{ch}$  are the vertical and horizontal clear edge distances.

Having derived the equations for each the of the connection capacities based on each of the

limit states, we can now obtain the connection capacity, as follows:

Assume plate thickness = 1/4"  
 Bolt shear strength = 44.2 kips  
 Horizontal edge distance = Vertical edge distance = 1.5"  
 Eccentricity = 3"  
 Bolt spacing = 3"  
 Fu = 58 ksi

The maximum capacity is found when  $\eta = 0$ . The individual limits are as follows:

$$\text{Capacity}_{bs} = \frac{26.1}{\sqrt{0^2 + \left(\frac{3}{3}\right)^2}} = 26.1 \text{ kips}$$

$$\text{Capacity}_{bs} = \frac{26.1}{\sqrt{(1-0)^2 + \left(\frac{3}{3}\right)^2}} = 18.5 \text{ kips}$$

$$\text{Capacity}_{to-top} = 0.75(1.5)(58)(0.25) \left[ \frac{1.5}{0} - \frac{1.0625}{2\sqrt{\left(\frac{3}{3}\right)^2 + 0^2}} \right] = \infty$$

$$\text{Capacity}_{to-side} = 0.75(1.5)(58)(0.25) \left[ \frac{1.5}{1-0} - \frac{1.0625}{2\sqrt{\left(\frac{3}{3}\right)^2 + (1-0)^2}} \right] = 18.3$$

Therefore the capacity of the connection is the minimum value of 18.3 kips. AISC in Table 10-9 reports the capacity of this connection as 23.0 kips. Using  $\eta = 0.5$  which maximizes the bolts results in a capacity of 16.7 kips.

It should be noted that when checking bearing and tearout a hybrid of equations J3-2a and J3-2b in the form of  $R_n = 1.5L_c t F_u \leq 2.4dt F_u$  is employed. This approach has been used for several reasons, First, the weak plate/ strong bolt model is predicated on the Lower Bound Theorem of Limit Analysis which guarantees that the applied external forces in equilibrium with the internal force field are less than or, at most, equal to the applied external force that would cause failure, provided that all the limit states are satisfied. This theorem is, strictly speaking, only valid as long as the connection is sufficiently ductile to allow redistribution of the forces. The use of the 1.5 factor is consistent with this approach since considerable deformations will occur as the forces in the bolts are distributed. Also since this is a weak plate/strong bolt model, the use of the 1.5 factor helps to minimize the thickness of the plate, thereby reducing the likelihood of bolt fracture and increasing the ductility of the connection. Further, the instantaneous center method itself is based on an assumption that rather large

deformations will occur in the connection. The load deformation curve which is used to determine the force at each bolt is based on a maximum, and relatively large, deformation of 0.34" for a 3/4" diameter A325 bolt, which tends to uphold the use of the 1.5 factor for bolt tearout. The  $2.4d_tF_u$  portion of the hybrid equation is included in an attempt to allow some deformation and redistribution of loads without cause excessive deformations. Even though a few bolts are allowed a capacity based on the larger deformation, the remaining bolts designed to the 2.4 bolt bearing equation, will tend to limit the overall deformation of the connection. Finally, bolt plowing is assumed to occur in a standard single plate shear connection, which is also consistent with the use of the 1.5 factor.

Table 1. Connection capacities using various models.

Bolts: 1" A490-X		Plate: 1/4" A36		Eccentricity = 3"		Edge Distance = 1 1/2"	
	AISC Table 10-9	Instantaneous Center Method w/ Eq. J3-2a	Instantaneous Center Method w/ Hybrid Eq. J3-2a & J3-2b	Proposed Method	Instantaneous Center Method Bolt Capacity		
(a)	23.0	13.4	16.7	18.3	38.8		
(b)	36.7	26.2	32.8	33.5	78.5		
(c)*	48.9	41.3	51.6	51.6	127		
* Net shear is the controlling limit state for this connection, so the capacity is 48.9 kips.							

It appears from Table 1 that the greatest benefit is achieved for cases with fewer bolts, this is because the horizontal component is larger and the bolt tearout becomes more critical. Of course as the eccentricity increases, the weak plate/strong bolt model becomes more advantageous to larger number of bolts as well. The model is also suited to analyze connections subjected to simultaneous axial and shear loads. A 1/4" plate was chosen for this example because it can be easily compared to the existing values presented in AISC Table 10-9. However it is not just the connection plate which can be the critical element, and some beam webs are thinner than 1/4". In such cases the weak plate/strong bolt model can be used to optimize the connection to even greater advantage.

One concern that arises is that the weak plate/strong bolt model does not take into account the load-deformation behavior of the bolts. It is assumed that the load can redistribute freely to optimize the connection. This is not a major concern for the simple cases shown here, because when the bolt shear and not plate bearing or tearout governs the capacity, the capacity is consistent with the instantaneous center method, which incorporates the load-deformation behavior of the bolts. However for more complicated geometries the capacity should be limited to that obtained using the instantaneous center method.

## CONCLUSION AND FURTHER WORK REQUIRED

As has been shown, the weak plate/strong bolt model can be used to mitigate the effects of the new AISC bearing and bolt tear-out requirements. However because of the nature of the problem, the number of equations to be solved increases exponentially as the number of bolts involved increases. The authors have developed ad hoc solutions to the three cases shown in Figure 2, as well as two further cases involving both shear and axial loads. However a non-linear programming solution still needs to be developed to increase the usefulness of the model for increased number of bolts with larger eccentricities.

## NOTATION

t	= the plate thickness
$F_u$	= the plate strength
d	= the bolt diameter
$e_i^2 = e_i \bullet e_i$	= clear edge distance
n	= the number of bolts
$R_{bs}$	= bolt shear constraint
$\eta$	= percentage of the vertical force resisted by the first bolt
s	= bolt spacing
$e_x$	= the eccentricity
$\phi_h$	= hole diameter
$L_{cv}$	= vertical clear edge distance
$L_{ch}$	= horizontal clear edge distance

## REFERENCES

1. AISC, 2001, Load and Resistance Factor Design, 3rd ed., American Institute of Steel Construction, Chicago, Illinois, pages 7-7,8 & 7-38 to 7-85

