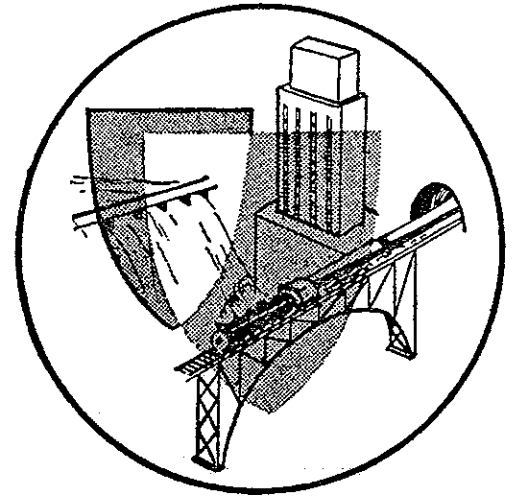


AND 2 1996

D. W. GOODPASTURE



EXPERIMENTAL INVESTIGATION OF STRESSES IN GUSSET PLATES

by R. E. Whitmore
Formerly Assistant Professor of
Civil Engineering
The University of Tennessee

BULLETIN NO. 16

MAY, 1952

ENGINEERING EXPERIMENT STATION
THE UNIVERSITY OF TENNESSEE
KNOXVILLE

***EXPERIMENTAL INVESTIGATION OF
STRESSES IN GUSSET PLATES***



R. E. WHITMORE

**Formerly Assistant Professor of
Civil Engineering
University of Tennessee**

ACKNOWLEDGEMENTS

The tests reported herein were carried on as a part of the research program of the Civil Engineering Department of The University of Tennessee. Professor Armour T. Granger, Head of the Department, has for many years been much interested in the particular problem of stresses in gusset plates, and this investigation has been carried out at his suggestion and under his supervision.

The first gusset plate study at The University of Tennessee was performed for a Master's thesis by F. J. Perna^{1*} in 1941, in which he used the photoelastic method to study stresses in a gusset plate. As the members and plate in this study were cut from a single sheet of bakelite, it was not clear what the effect of connecting the members to the plate by rivets would have been on the indicated stresses. This led to further testing by J. A. Sandel² and the writer³ in 1950, the work being done as the basis for Masters' theses. Further refinements were made in the model used by the writer, and with the invaluable assistance of B. F. Goldhammer, then Instructor in Civil Engineering, the additional tests reported herein were performed as a research project of the Civil Engineering Department.

Special acknowledgement and thanks are due to Professor W. M. Murray, Massachusetts Institute of Technology, Editor of the Proceedings of the Thirteenth Semi-annual Eastern Photoelasticity Conference, for permission to quote from the article "Steel Gusset Plates" by R. H. Rust. Similar acknowledgement is owed to John Wiley and Sons for their courtesy in permitting use of the quotation from "Structural Design in Steel" by T. C. Shedd.

Acknowledgement is also made to O. I. Clift, mechanic of the Civil Engineering Department, for help in constructing the model and loading frame, and A. O. Webb, formerly Assistant Professor of Drawing, for the preparation of the drawings. Professor C. R. Walker assisted by checking some of the computations.

*Superscript numbers refer to references at end of text.

CONTENTS

Summary	1
Introduction	2
Preliminary Statement	2
Purpose of Investigation	3
Scope of Investigation	3
Prototype and Model	3
The Prototype	3
The Model	5
Materials and Apparatus	5
Model Materials	5
Strain Measuring Devices and Equipment	6
The Loading System	8
Application of Loads	8
Measurement of Loads	9
Testing Procedures	11
Strain Gage Method	11
Stresscoat Method	11
Results	12
Strain Gage Tests	12
Distribution of Stresses	15
Stresscoat Tests	19
Validity of Results	20
Comparison of Results Obtained by Different Experimental Methods	20
Comparison with Analytical Results	21
Statistical Checks	27
Conclusions	29
References	33

Experimental Investigation of Stresses in Gusset Plates

SUMMARY

This bulletin describes an experimental investigation which was performed to determine the stress distribution in certain types of gusset plates, including the maximum intensity of stress and its location, and to devise a simple method of determining approximately the maximum stresses, for use in structural design.

To accomplish these objectives, experiments were performed on models of aluminum, masonite and bakelite, using wire-bonded strain gages, brittle lacquers and photoelastic procedures. Joint L_2 of a Warren truss served as the prototype.

For the joint tested, the maximum tension was found to be near the end of member U_1L_2 (the tension diagonal) and the maximum compression near the end of L_2U_3 (the compression diagonal). Maximum shearing stress was located close to the top of the chord and near the center of the plate width. Edge stresses were found to be much less serious than expected and the straight-line distribution of stress customarily assumed in analyzing such plates was found to be incorrect.

Based on these tests, it was found that maximum tensile and compressive stresses may be approximated quite closely by assuming the force in each diagonal to be uniformly distributed over an area obtained by multiplying the thickness of the plates by an effective length normal to the axis of the member. This effective length is obtained by constructing lines making angles of 30 degrees with the axis of the member which originate at the outside rivets in the first row and continue until they intersect a line perpendicular to the member through the bottom row of rivets; the effective length is the intercept on this line between the two inclined lines.

The maximum shearing stress can be determined with sufficient accuracy by dividing the total horizontal shear on the plate (the sum of the horizontal components of the diagonals) by the area of the gusset plates on a horizontal cross-section, and multiplying the result by $1\frac{1}{2}$. This agrees with the usual or conventional method of computing shearing stress in such plates.

INTRODUCTION

Preliminary Statement.

Many years have passed since the designing of pin-connected trusses was virtually discontinued. The now familiar truss with members riveted to gusset plates has gained increasing popularity, due largely to its inherent strength, stiffness and rigidity. It seems a reasonable assumption that rivets (or bolts) are used for perhaps 90 per cent of all connections in present structural practice as applied to trusses.

Gusset plates are an integral part of structural steel frames. They are fundamental to the continuity of the structure, and therefore merit careful consideration. Frequent and elaborate analyses have been made concerning other structural elements, with the result that they may be designed with a high degree of engineering accuracy. The plates, however, which must transfer the stress between the members, have not been subjected to such diligent investigation and therefore represent to some extent a discontinuity in the knowledge of stresses in riveted structures.

Important as the problem may be, a survey of published material shows that there have been relatively few attempts to determine experimentally the distribution of stresses in gusset plates. Modern practice generally is to make the size of the plate in the plane of the truss as large as necessary so that all required rivets may be placed properly, and to select a tentative thickness in accordance with values commonly used for similar trusses. The stresses are then analyzed in some manner to insure the plates being adequate and economical. Such an analysis generally consists of passing various sections through the plate and solving for direct, bending, and shearing stresses, thus necessitating the making of certain questionable assumptions.

Because of the apparent uncertainty as to the stress distribution and intensity of stress, and lack of knowledge as to how to determine these analytically, experimental investigations appear highly desirable. In fact T. H. Rust⁴ has said:

"It is difficult to believe that there is a more important or more fundamental problem in need of further investigation in the field of structural engineering than steel gusset plates. They constitute a formidable problem in stress analysis capable of further exploitation in the laboratory . . ."

Purpose of Investigation

The need of definite information regarding stresses in gusset plates prompted the experimental investigations described herein. Specifically, the purpose of this research was to:

- a. Determine the general stress distribution in gusset plates,
- b. Determine the maximum intensity of stress and its location,
- c. Devise a convenient method of determining the maximum stresses with sufficient accuracy to use in designing structures.

Scope of Investigation

An investigation of stresses in any single gusset plate will not provide a general solution for all gusset plates. A solution determining the location and magnitude of the maximum stress for a general case (if there is such a case) would necessitate an expensive and long-range program of research.

To expedite matters, this initial investigation was restricted to one particular type of gusset plate. To simplify the analysis further, the effect of only direct stresses (i.e., primary stresses) in the members was considered.

Unquestionably, additional supplementary investigations would be desirable. These could deal with other types of plates, plates for other truss types, and the investigation of the effect of bending (i.e., secondary stresses). However, it is felt that some of the results of this particular investigation will have general application to all types of gusset plates and will possibly stimulate additional work in this field.

PROTOTYPE AND MODEL

The Prototype

Inasmuch as the present study was to be limited to one particular type of plate, and since no single type could be completely representative, it was desired that the one selected represent a large and important group of plates. This suggested a joint in a common type of truss, with several members connected, but with no chord splice in the joint.

Joint L_2 of a through Warren type highway bridge was selected, since the Warren truss is the usual—almost universal—truss type used in modern bridge design. L_2 is typical of almost all bottom joints where the chord is straight, and the results obtained should be directly applicable to a great many joints.

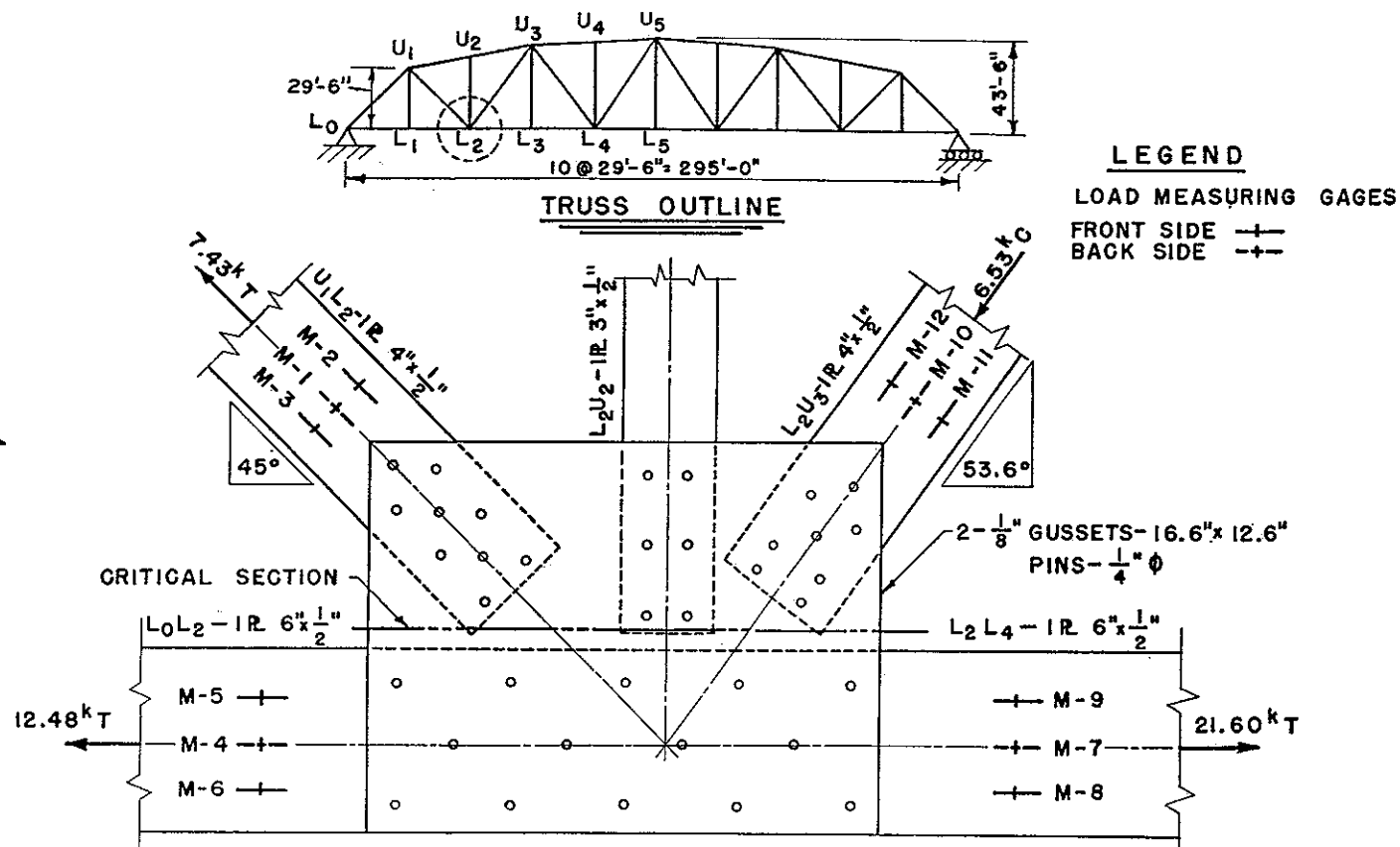


Figure 1. The Model—Joint L_2 of Warren Truss.

The Model

Since it was not practicable to test a full scale model, a small model of the size shown in Figure 1 was used. At a scale ratio of 1:4, this represents a prototype of two plates 66 inches by 50 inches by $\frac{1}{2}$ inch. This is typical of the plates at this joint for a highway truss span of approximately 300 feet having a two-lane roadway with concrete floor, and designed for an H-20 loading.

The exact shape of the members was thought to be relatively unimportant for this problem; hence solid rectangular sections of sufficient cross-sectional area were selected for the members for convenience in constructing and testing the model.

A simple, symmetrical arrangement of pins and bolts was used. The diameter of $\frac{1}{4}$ inch represents a prototype rivet of one-inch diameter. To assure unity of action of the two separate plates and to simulate the action of the rivets in the prototype, approximately one third of the holes were filled with tight-fitting bolts and the remainder with tight-fitting pins. The contact between the plates and members allowed some of the load to be transferred by friction, which is undoubtedly similar to the action in a real truss.

In spite of the size reduction, arrangement of rivets, and other simplifications, it is believed that the stresses in the model are definitely representative of stresses which would occur in the prototype.

MATERIALS AND APPARATUS

Model Materials

The fundamental consideration in the selection of the model material was the fact that the experimental method to be used would be strain measuring. Consequently, large strain magnitudes in the plate were desired, thus requiring a material with a low modulus of elasticity. Also, it was desired that the material be of reasonably high strength, light in weight, ductile, and in general possessed of properties similar to those of structural steel. All of these considerations indicated that a high strength aluminum alloy would be the most suitable material. The aluminum alloy selected was 61-ST, which has the following physical and mechanical properties (average values):

Modulus of elasticity	10,000,000 psi
Modulus of rigidity	3,800,000 psi
Poisson's ratio	0.33
Yield strength (tension and compression)	39,000 psi

Ultimate tensile strength	45,000 psi
Per cent elongation in two inches	15 per cent
Yield strength (shear)	26,000 psi
Ultimate strength (shear)	30,000 psi
Weight	0.098 lb per cu in

Alloys:

Copper	0.25 per cent
Silicon	0.60 per cent
Magnesium	1.00 per cent
Chromium	0.25 per cent

This alloy is recommended for structural uses where good strength and resistance to corrosion are desired⁵.

The plates, members and pins were all of the same alloy, while the bolts were tight-fitting aircraft bolts of aluminum alloy, exact composition unknown.

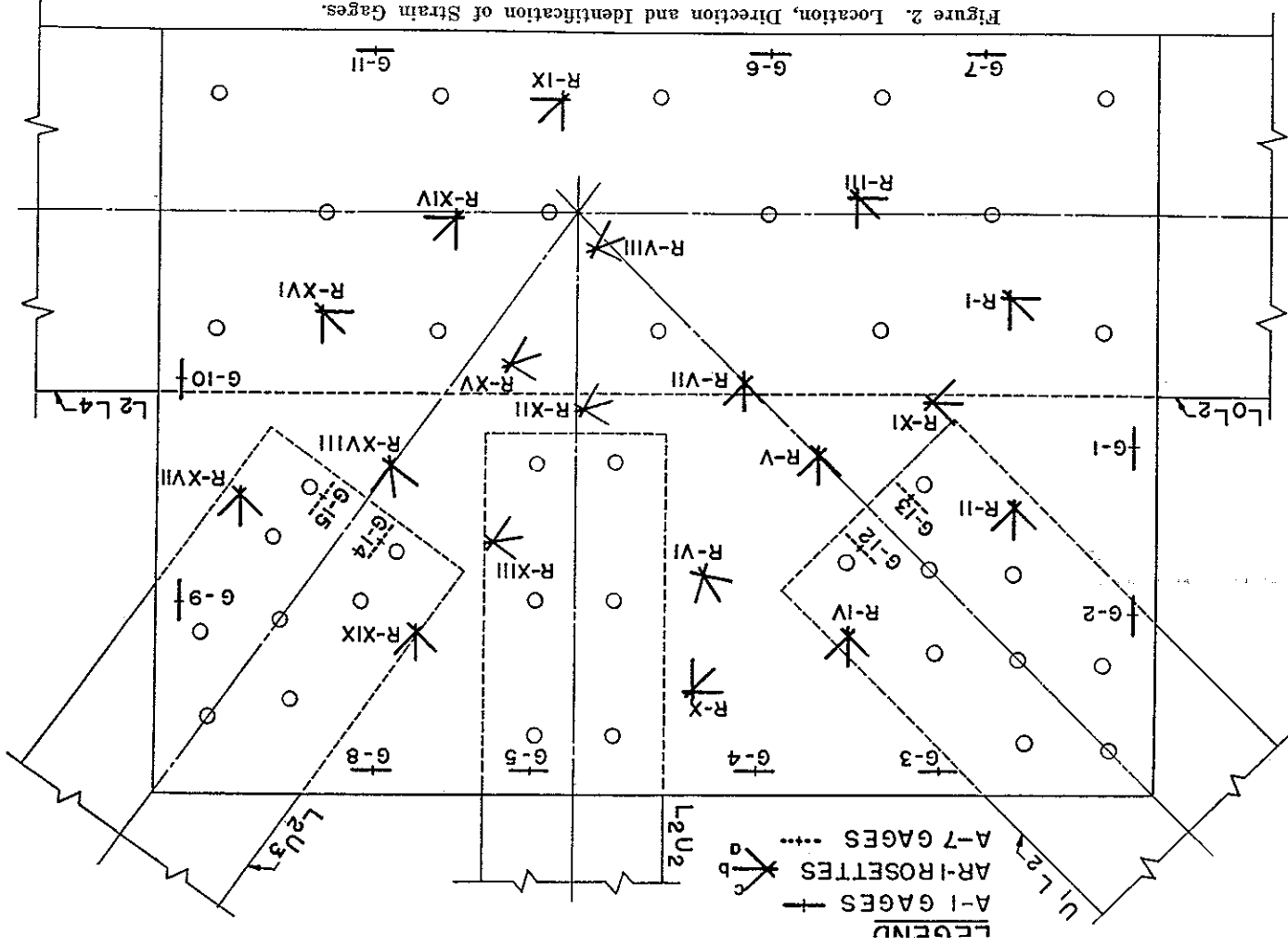
Strain Measuring Devices and Equipment

The principal method of measuring strains was by the use of SR-4 wire bonded strain gages. So that the complete picture of the state of stress within the plates would be known, it was necessary to determine the principal stresses and their directions (p , q , and θ). Strain rosettes were used for this purpose except at the edges of the plate. Rosettes available are of several types, but a simple 45-degree rectangular rosette (type AR-1) with three gages at 45 degrees with one another was selected. For a given gage length, this type of rosette covers a minimum area.

Along the edges of the plate, the direction of the principal stresses, and the magnitude of one of the stresses, are known. Therefore it was unnecessary to use costly rosettes along the periphery, and single gages of type A-1 were used here.

Nineteen AR-1 rosettes and eleven A-1 gages were cemented to each plate, their locations being such that they were well scattered but more closely spaced in what were expected to be critical regions. Furthermore, it was endeavored to place the gages with their axes approximately in the direction of the principal stresses. The location and direction of the gages are shown in Figure 2.

This pattern was exactly duplicated on each of the two plates at the joint, so that at each gage location two independent gage readings would be available. The purpose of this arrangement was to enable the two readings to be averaged and thus to eliminate



the effect of bending perpendicular to the plane of the truss on the stresses and strains in the plates.

On one plate only, four very small gages (type A-7) were attached adjacent to the rivets at the ends of the diagonals (see Figure 2). The purpose of these gages was to determine the effect of stress concentration near the holes in the plate.

Flexible, stranded copper wire was used to connect the gages to the indicator used to measure strains. So that each gage could be connected permanently throughout the series of tests, each gage was connected to a switch. The switches used were single-circuit with twenty-four positions. As there were 152 gages in all, seven switches were required. The switches were connected through a common terminal to a type "K" portable strain indicator.

To give additional information and verification, two plates, of the same size (except for thickness) and shape as the aluminum plates, were made from fiberboard. These plates were sprayed with a brittle lacquer which will crack in tension when the strain in the plate is such that the ultimate tensile strength of the coating is exceeded. The material used in this study was "Stresscoat."

THE LOADING SYSTEM

Application of Loads

It was realized in the early planning stages that applying axial loads of the desired magnitudes would be a major problem. The fact that the loads would be as large as 20,000 pounds or more, and that five members with four different directions were involved, indicated the complexity of the problem. Consequently, a very simple type of loading arrangement was deemed necessary.

The initial simplification was to make the stress in member L_2U_2 zero. This member in a Warren truss normally carries only a small fraction of the loads in the other members, so it seemed reasonable to neglect it.

A rectangular frame (Figure 3) large enough to accommodate the model was constructed of 2½-inch standard steel pipe. Cross-members perpendicular to the diagonals were welded to the frame for the purpose of applying loads to the diagonal members. Later testing indicated that excessive bending of this frame resulted when the model was under full load, and it was reinforced by the welding of steel plates 3½ inches by 3½ inches to it.

The manner in which tensile or compressive forces were applied to the members should be apparent from a consideration of Figures

3 and 4. The model was attached to the loading frame by specially built clevises, one at the end of each member. A clevis consisted essentially of two plates which at one end engaged the member through a transverse pin and at the other were welded to a threaded rod provided with suitable nuts. Tension or compression was applied by tightening and loosening the appropriate nuts so as to move the clevis in the proper direction. To eliminate transverse and longitudinal bending in the members, each clevis was equipped with an adjusting ring providing the necessary movements by means of four set screws, and a ball thrust bearing to reduce friction.

Measurement of Loads

It was originally planned to predetermine the exact axial load to be applied to each member. To measure these loads, three SR-4 type A-1 gages were cemented to each member to be loaded. One gage was located on the central axis of the member on one face and two gages, spaced equidistant from the axis, were located on the opposite face. All gages were placed parallel to the axis of the member. With this arrangement, bending in the plane of the truss, and also perpendicular to it, could be detected. These gages were connected to the switching arrangement and thence to the strain indicator in the manner previously described.

With zero load applied to the members, the indicator was read for each gage. The indicator reading each gage should have at full load could then be calculated by adding algebraically the proper computed unit strain to the original indicator values.

Due to problems arising in the laboratory, this procedure could not be followed in detail. The exact predetermined gage readings could not be obtained simultaneously, since in adjusting the gage readings for one member, all other gage readings were somewhat affected. Furthermore, bending perpendicular to the plane of the truss could not be entirely eliminated by means of the adjusting screws. Therefore, the procedure adopted was to load the members until all gages read approximately the desired values, to reduce the bending to a minimum by making all possible adjustments, and then to calculate the load in each member by converting the average strains to average stresses and then to loads.

These calculated loads were then adjusted to give equilibrium of the system; i.e., $\sum H=0$ and $\sum V=0$. The data and results of this procedure for two laboratory tests are given in Table 1.

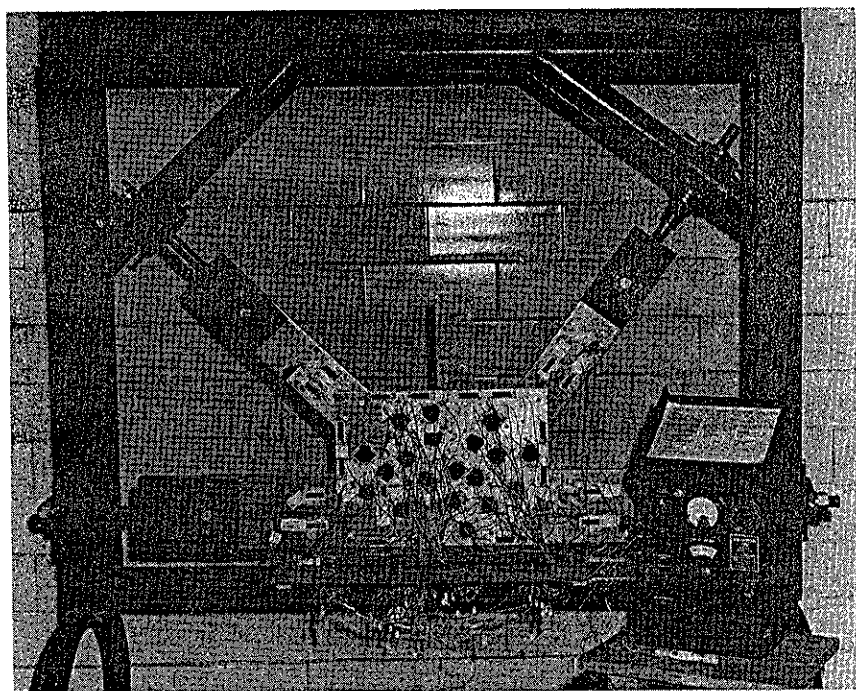


Figure 3. General View of Experimental Apparatus.

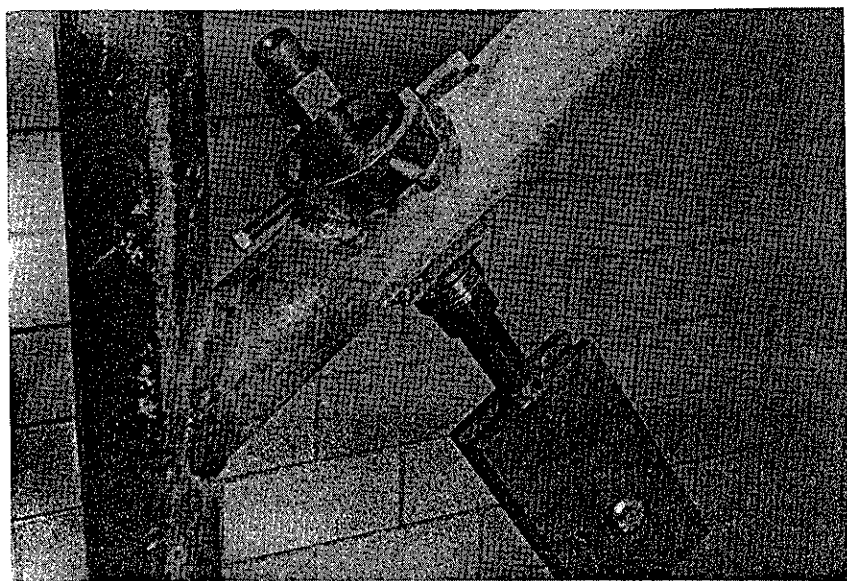


Figure 4. The Loading Device.

TABLE 1

Strains and Calculated Loads in Members of the Model^a

<i>Member</i>	<i>Gage No.</i>	<i>Unit Strain Test No. 1 (micro-in per in)</i>	<i>Unit Strain Test No. 2 (micro-in per in)</i>	<i>Ave. of Tests 1 & 2 (micro-in per in)</i>	<i>Ave. Unit Strain On Cross-Section (micro-in per in)</i>	<i>Calculated Loads (pounds)</i>	<i>Adjusted Loads (pounds)</i>
10	U ₁ L ₂	M-1	+130	+120	+125	$\frac{610+125}{2}$	2x10x368
		M-2	+605	+595	+600	2	= 7,350 lb T
		M-3	+620	+615	+620	= +368	7,380 lb T
	L ₀ L ₂	M-4	+300	+295	+300	$\frac{565+300}{2}$	3x10x433
		M-5	+570	+580	+575	2	=13,000 lb T
		M-6	+550	+560	+555	= +433	12,600 lb T
	L ₂ L ₄	M-7	+500	+500	+500	$\frac{920+500}{2}$	3x10x710
		M-8	+920	+925	+925	2	=21,300 lb T
		M-9	+910	+915	+915	= +710	21,600 lb T
	L ₂ U ₃	M-10	-375	-375	-375	$\frac{-315-375}{2}$	2x10x345
		M-11	-305	-290	-300	2	= 6,900 lb C
		M-12	-325	-335	-330	= -345	6,530 lb C

^a"Plus" strains correspond to tensile stresses and "negative" strains to compressive stresses.

It will be noted that the residual bending after all adjustments were made is primarily perpendicular to the truss. This bending tended to reduce the stress in one plate and increase the stress in the opposite plate. As gages identically located were attached to each plate, and the average of the two readings used in later calculations, it is believed that the effect of such bending has been reduced to a negligible amount, if not altogether eliminated.

TESTING PROCEDURES

Strain Gage Method

The nineteen AR-1 rosettes and eleven A-1 gages were cemented to each plate following recommended procedure. Each gage was identified by number, and the exact location and alignment determined. This information is itemized in Table 2, together with the gage factor of each gage.

Each gage was connected to the strain indicator through the switching arrangement and a "no load" indicator reading taken for each gage. After the loads had been applied to the members, the indicator readings were again recorded for each gage. The strain for each gage was obtained by subtracting the "no load" from the loaded indicator readings, the units being micro-inches per inch.

This procedure was repeated for decreasing load and, later, the entire procedure repeated for increasing and decreasing loads, thus giving four independent readings. The maximum difference between readings of any one gage was small and apparently was due to the fact that the loads could not be reproduced exactly for each test. This reproducibility of strain values indicates that all gages were operative and that the readings could be used to calculate stresses with sufficient accuracy.

Stresscoat Method

Before the strain gages were attached, a stresscoat test of the aluminum plates was attempted. However, because the strains in the plates were of low magnitude, the test was not successful.

Later, plates of the same size and shape made of fiber-board (Masonite) were substituted for the aluminum plates. Although this material is neither perfectly homogeneous nor isotropic, it is believed that for purely qualitative information it will give reasonably reliable results. Masonite was chosen because of its low modulus of elasticity, because it could be easily cut to the desired size and shape, and because of its smooth surface.

After fabrication, the members were attached and the assembly placed in the loading frame. The proper lacquer for the temperature and humidity was then sprayed on each plate. Loads were slowly applied to each member and an attempt was made to maintain the same ratio of loads between the members as had been used in the strain gage test.

As the tension cracks appeared, their location was noted and a sketch made. This procedure was continued until failure of the plates occurred.

TABLE 2
Location, Direction, and Gage Factor of
Gusset Plate Gages

Gage No.	Coordinates ^a		Direction ^b (degrees)	Gage Factor	
	X (inches)	Y		Front Plate	Rear Plate
G-1	0.38	6.75	90	2.07	2.03
G-2	0.38	9.53	90	2.07	2.03
G-3	3.56	12.16	0	2.07	2.03
G-4	6.61	12.16	0	2.07	2.03
G-5	10.40	12.16	0	2.07	2.03
G-6	6.46	0.38	0	2.07	2.03
G-7	2.90	0.38	0	2.07	2.03
G-8	13.02	12.16	0	2.07	2.03
G-9	16.22	9.38	90	2.07	2.03
G-10	16.22	5.77	90	2.07	2.03
G-11	13.04	0.38	0	2.07	2.03
R-I	2.46	4.41	90	2.07	2.03
R-II	2.30	7.78	47	2.07	2.03
R-III	5.08	2.82	92	2.07	2.03
R-IV	5.08	9.88	44½	2.07	2.03
R-V	5.59	6.96	45	2.07	2.03
R-VI	7.46	8.95	76	2.07	2.03
R-VII	6.86	5.67	60½	2.07	2.03
R-VIII	8.80	3.61	154	2.07	2.03
R-IX	9.94	1.11	0	2.07	2.03
R-X	7.67	10.89	179	2.03	2.03
R-XI	3.69	6.10	135	2.03	2.03
R-XII	9.52	6.13	148	2.03	2.03
R-XIII	11.00	8.32	143½	2.03	2.03
R-XIV	11.71	3.09	0	2.07	2.03
R-XV	10.63	5.54	148½	2.07	2.03
R-XVI	13.86	4.68	90	2.07	2.03
R-XVII	15.20	7.58	53½	2.07	2.03
R-XVIII	12.66	7.09	53½	2.07	2.03
R-XIX	12.24	9.74	53½	2.07	2.03

^aOrigin of coordinate axes is the lower left hand corner of gusset plate.

^bAngle given is from horizontal counterclockwise to the "a" component of the rosette gages.

RESULTS

Strain Gage Tests

A summary of the gage readings is given in Table 3. The data are the averages of gages on both plates for two laboratory

tests—increasing and decreasing loads. The calculated strains are also given, plus strains corresponding to tensile stresses and negative strains to compressive stresses. The forces in the members compatible with these data are given in Table 1.

These strains in themselves are of little value, since the primary interest is in the value of stress. The usual conversion of unit strains to stresses is by calculating principal strains and then converting them to principal stresses.

For the edge gages, the calculation of stresses was elementary, since the product of modulus of elasticity and gage reading (unit strain) resulted in the principal stress (the other being zero).

TABLE 3
Summary of Strains from Readings of
Gages on the Model^a

Gage No.		Ave. Strain— Both Plates Test No. 1 (micro-in per in)	Ave. Strain— Both Plates Test No. 2 (micro-in per in)	Ave. Strain— Both Plates Tests 1 and 2 (micro-in per in)
R-I	a	80	80	80
	b	160	170	165
	c	-20	0	-10
R-II	a	-130	-120	-125
	b	250	240	245
	c	265	265	265
R-III	a	5	20	15
	b	185	200	190
	c	120	140	130
R-IV	a	-120	-110	-115
	b	-75	-80	-80
	c	250	250	250
R-V	a	-275	-270	-270
	b	75	80	80
	c	450	450	450
R-VI	a	-85	-130	-110
	b	200	190	195
	c	200	205	200
R-VII	a	-250	-250	-250
	b	175	190	180
	c	370	390	380
R-VIII	a	310	330	320
	b	-40	-20	-30
	c	-240	-240	-240
R-IX	a	190	180	185
	b	-30	-45	-40
	c	-60	-50	-55
R-X	a	80	60	70
	b	-70	-90	-80
	c	-25	-40	-35

TABLE 3 (Continued)
Summary of Strains from Readings of
Gages on the Model^a

Gage No.		Ave. Strain— Both Plates Test No. 1 (micro-in per in)	Ave. Strain— Both Plates Test No. 2 (micro-in per in)	Ave. Strain— Both Plates Tests 1 and 2 (micro-in per in)
R-XI	a	395	360	380
	b	25	10	20
	c	-155	-155	-155
R-XII	a	390	360	375
	b	-40	-45	-40
	c	-330	-335	-330
R-XIII	a	220	205	210
	b	-150	-140	-145
	c	-320	-320	-320
R-XIV	a	140	160	150
	b	-170	-155	-165
	c	-60	-55	-60
R-XV	a	430	420	425
	b	-15	-10	-10
	c	-385	-365	-375
R-XVI	a	-140	-165	-155
	b	130	150	140
	c	115	165	140
R-XVII	a	-280	-280	-280
	b	-250	-230	-240
	c	70	90	80
R-XVIII	a	-450	-440	-445
	b	-95	-100	-100
	c	320	330	325
R-XIX	a	-210	-210	-210
	b	110	120	115
	c	90	100	95
G-1		130	140	135
G-2		370	370	370
G-3		250	240	245
G-4		70	70	70
G-5		-65	-60	-65
G-6		105	130	120
G-7		0	0	0
G-8		-210	-230	-220
G-9		-370	-415	-390
G-10		-135	-155	-145
G-11		215	240	225

^a"Plus" strains correspond to tensile stresses and "negative" strains to compressive stresses.

The interior gages presented a more complex problem. However, three normal strains measured in known directions relative to one another can be converted mathematically to principal strains at that point. Various analytical, semi-graphical and graphical methods have been developed to accomplish this purpose. Principal

strains and their directions for each rosette were determined graphically.

Since the aluminum used in this study can be considered isotropic, the principal directions of stress and strain are coincident.

The relationship between the principal stresses and principal strains has been established by the theory of elasticity. It is known that

$$p = \sigma_{\max} = \frac{E}{1 - \mu^2} (\epsilon_{\max} + \mu \epsilon_{\min})$$

$$q = \sigma_{\min} = \frac{E}{1 - \mu^2} (\epsilon_{\min} + \mu \epsilon_{\max})$$

in which σ = unit normal stress

ϵ = unit normal strain

μ = Poisson's ratio

E = modulus of elasticity

Therefore, with the principal strains calculated and with the elastic constants known, it is a simple matter to calculate the principal stresses.

Tatnall⁶ has suggested a graphical method whereby the three strain readings for one rosette may be plotted to scale and the direction and magnitude of principal strains determined. This construction is essentially a Mohr circle. The stress circle can then be plotted with the same center by merely changing the scale of the radius by the factor $\frac{1 - \mu}{1 + \mu}$. Whatever scale is used for strain must be multiplied by $\frac{E}{1 - \mu}$ to determine the stress. This method was found to be most expedient.

As a check on the results, Lee's⁷ formulas were used for direct calculation of p , q , and θ from the three strain readings and the known elastic constants.

The maximum shear stress at each gage location was calculated by taking one half of the difference of principal stresses.

A summary of principal strains, principal stresses, their directions, and maximum shear stresses is given in Table 4.

Distribution of Stresses

The state of stress at a point can best be represented by the maximum and minimum normal stresses, the maximum shearing stress, and the direction of these stresses referred to some established reference axis.

TABLE 4
Principal Strains and Stresses, Their Directions
and Maximum Shear Stresses^a

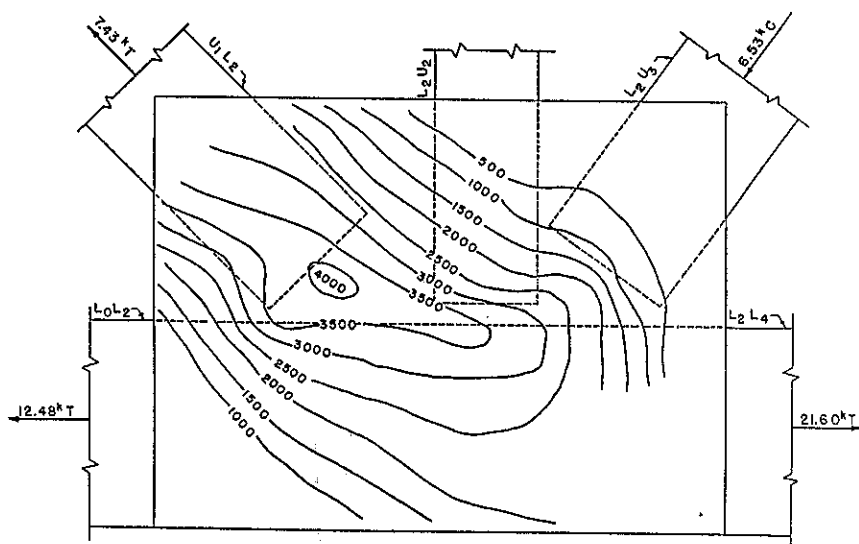
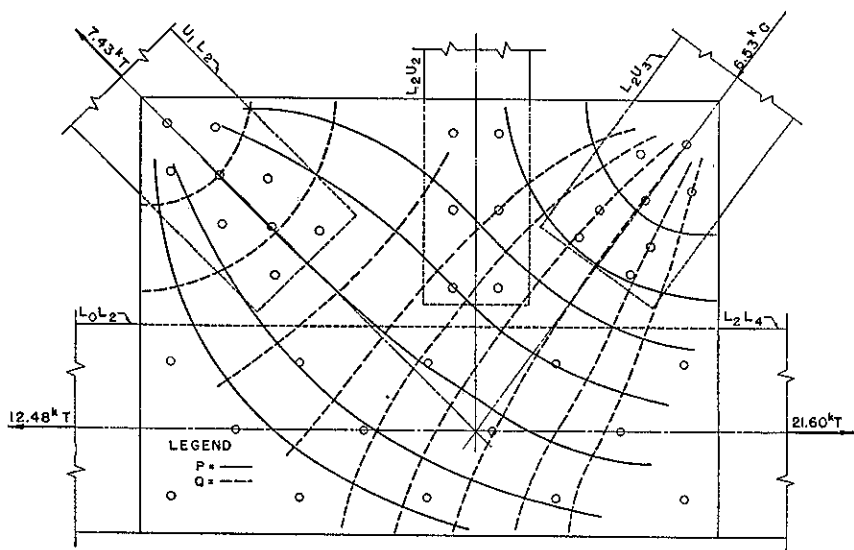
Gage No.	Principal Strains (micro-in per in)		Principal Stresses (lb per sq in)		Max. Shear Stress τ (lb per sq in)	Direction ^b θ (degrees)
	ϵ max.	ϵ min.	p	q		
R-I	173	-102	1,550	-510	1,030	125½
R-II	333	-193	3,010	-920	1,965	117
R-III	208	-58	2,120	+120	1,000	150
R-IV	304	-167	2,770	-760	1,765	154
R-V	450	-270	4,045	-1,355	2,700	136
R-VI	261	-171	2,300	-950	1,625	144
R-VII	400	-270	3,490	-1,550	2,520	140½
R-VIII	330	-250	2,765	-1,565	2,165	147
R-IX	225	-94	2,165	-225	1,195	159½
R-X	131	-94	1,090	-570	830	148
R-XI	390	-170	3,805	-445	2,125	125½
R-XII	382	-333	3,030	-2,310	2,670	143
R-XIII	227	-338	1,280	-2,920	2,100	134
R-XIV	280	-190	2,430	-1,090	1,760	148
R-XV	428	-377	3,390	-2,640	3,015	146
R-XVI	200	-214	1,455	-1,675	1,580	157½
R-XVII	130	-330	210	-3,210	1,710	162½
R-XVIII	327	-448	2,010	-3,800	2,905	146½
R-XIX	176	-285	865	-2,585	1,725	119½
G-1	135	0	1,350	0	675	90
G-2	370	0	3,700	0	1,850	90
G-3	245	0	2,450	0	1,225	0
G-4	70	0	700	0	350	0
G-5	0	-65	0	-650	325	90
G-6	120	0	1,200	0	600	0
G-7	0	0	0	0	0	0
G-8	0	-220	0	-2,200	1,100	90
G-9	0	-390	0	-3,900	1,950	0
G-10	0	-145	0	-1,450	725	0
G-11	225	0	2,250	0	1,125	0

^aValues given are average of both plates.

^bAngle given is from horizontal counterclockwise to the "p" direction.

To illustrate the findings of this research with regard to the distribution of stresses in the gusset plate, the following figures were sketched from experimental results:

- Stress trajectories (lines so drawn that their tangents at every point are in the direction of the principal stress) for p and q , Figure 5,
- Contours of maximum tensile stress, Figure 6,
- Contours of maximum compressive stress, Figure 7,
- Contours of maximum shearing stress, Figure 8,
- Isoclinics (contours of principal stress directions), Figure 9.



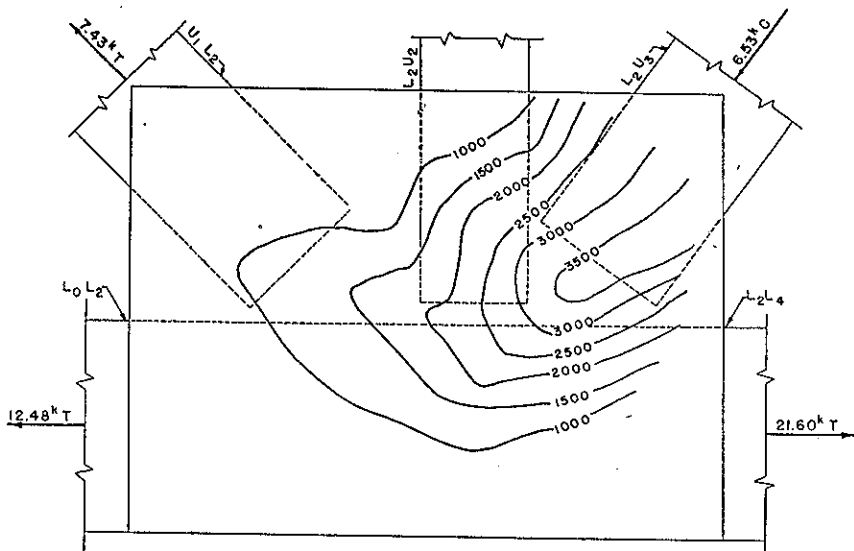


Figure 7. Contours of Maximum Compressive (q) Stresses, in Pounds per Square Inch.

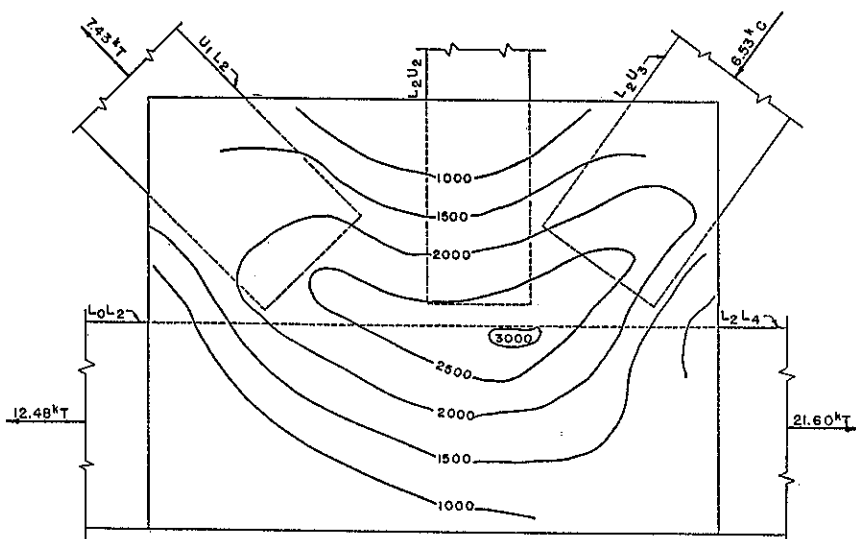


Figure 8. Contours of Maximum Shearing (τ) Stresses, in Pounds per Square Inch.

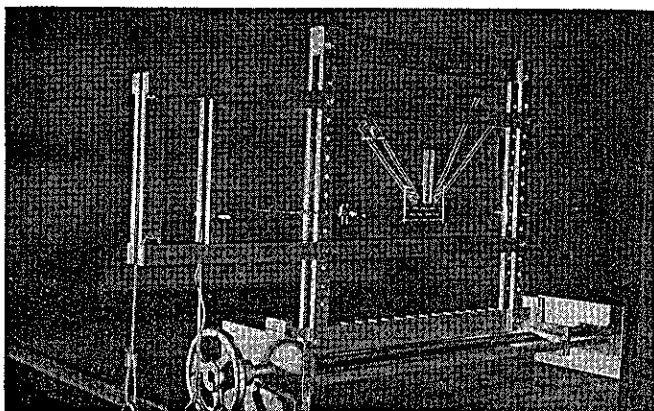


Figure 10. Transparent Bakelite Model in Loading Frame.

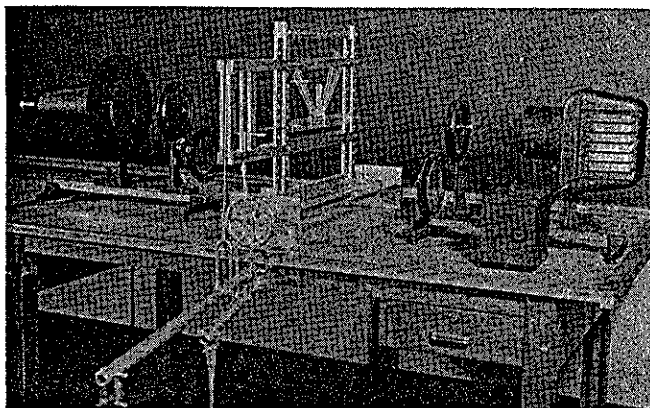


Figure 11. General Arrangement of Model and Polariscope.

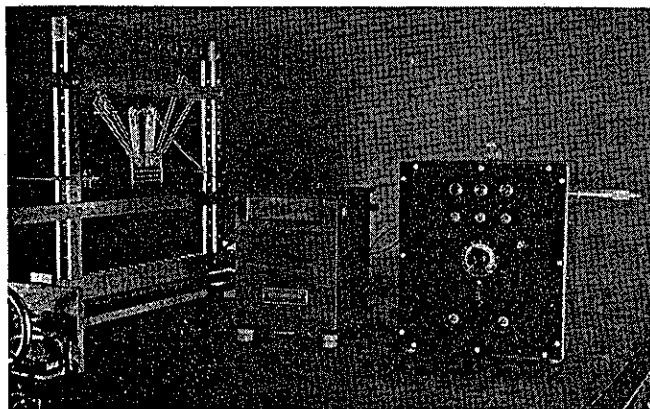


Figure 12. Model Equipped with Lateral Extensometer.

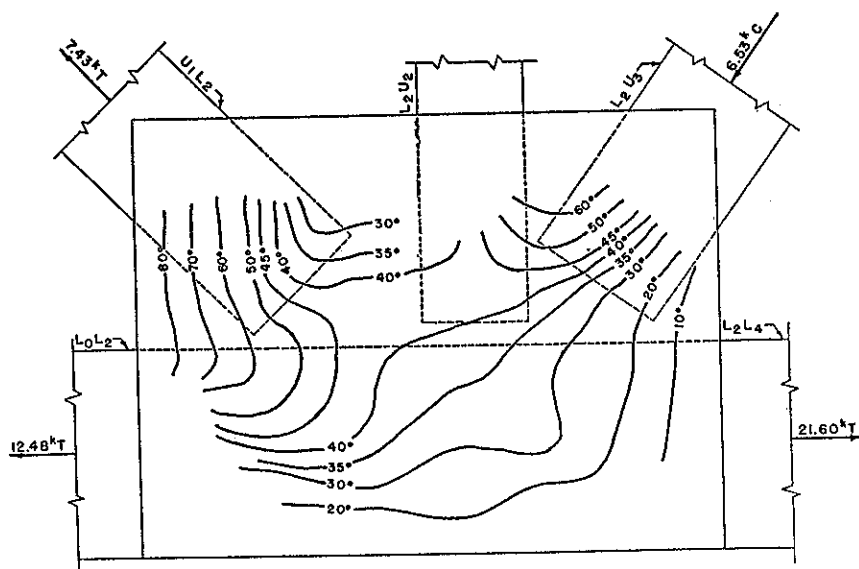


Figure 9. Isoclinics. Angles are Measured Clockwise from the Horizontal to the "p" Direction.

Due to insufficient coverage of the plate with gages in areas which were not considered critical, contours could not be sketched over the entire plate. Consequently, some of the plate area has been left blank. In addition, the sketches were made without consideration of the localized disturbance caused by rivets and rivet holes. It is known that a hole is a "stress raiser" and that the stresses in the immediate vicinity of the holes will be larger than those indicated by the contours.

The contours given in Figures 5 to 9 are to some extent approximate due to the lack of sufficient points to permit more exact interpolation and to the fact that the localized stresses at the holes were not considered. The contours do, however, represent the manner in which the stresses are distributed throughout the gusset plate in question and afford an excellent visualization of the location of the highly stressed regions.

Stresscoat Tests

No attempt was made to obtain quantitative results by the use of Stresscoat. The qualitative results, however, were quite definite. A sketch of the crack pattern obtained is given in Figure 13.

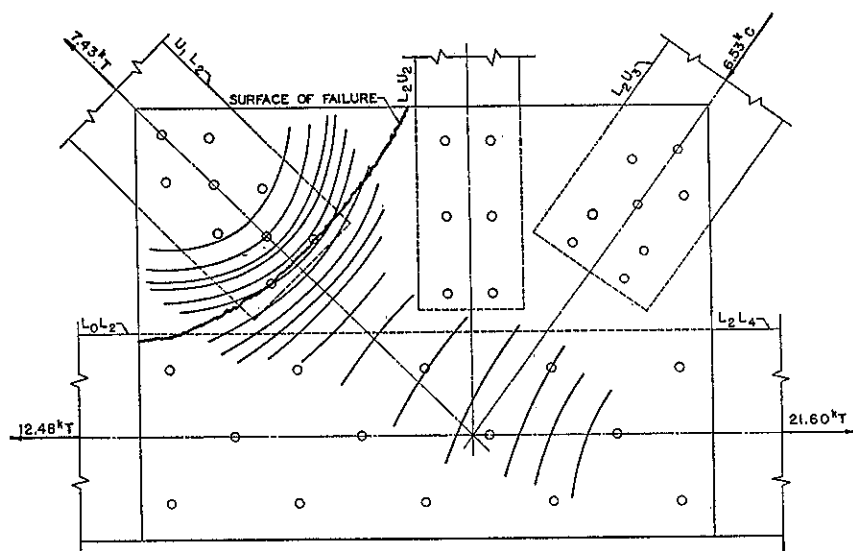


Figure 13. Tension Cracks in the Stresscoat.

Since the cracks are orthogonal to the direction of maximum tension, it is quite clear where the serious tensile strains are located. The cracks indicate that the tensile force travels a path from the upper left hand corner (U_1L_2) to the lower right side of plate (L_2L_4). This of course seems somewhat obvious when the nature of the forces is carefully considered.

The first cracks appeared near the end of the tension diagonal, indicating this to be the region of maximum tensile strain. The load was gradually increased until failure of the plate occurred. The surface of failure (shown in Figure 13) roughly corresponds to the "q" stress trajectory passing through the last row of rivets in the tension diagonal. Results of the test indicate that the maximum tensile stresses in the plate are near the end of the tension diagonal and parallel to the direction of this member.

VALIDITY OF RESULTS

Comparison of Results Obtained by Different Experimental Methods

It is of course desirable to check the validity of experimental studies by as many methods as practicable. Some independent verifications of the results stated above have been made by other methods, and are available for comparison. These are discussed briefly below.

The first of such checks, already presented, is the fact that different strain gage tests performed on the model resulted in almost identical stress patterns. Furthermore, a comparison of Figures 5 and 13 will illustrate the marked similarity of results obtained by the two different methods used. The cracks in the Stresscoat are of course stress trajectories for "q" and they closely parallel the "q" stress trajectories given in Figure 5. Also, both the close spacing of the cracks and the location of the plane of failure indicate that the maximum tensile stresses are near the end of the tension diagonal. This verifies similar results obtained by the use of strain gages.

As was briefly mentioned in the Introduction, other experimental studies, by means of the photoelastic method, have been performed at The University of Tennessee, the most significant of these being the basis of a Master's thesis by J. A. Sandel². In this study, Sandel investigated the stresses in a bakelite model of a gusset plate similar in detail to the larger aluminum model used by the writer. One difference, necessary in the use of the photoelastic method, was that one plate with double plane members was used rather than two plates with single plane members. Sandel used a polariscope with both white and monochromatic light to obtain the stress difference, $p - q$, by the usual method. To obtain the stress sum, $p + q$, he used a highly sensitive lateral extensometer. The general arrangement, including model, loading frame, polariscope and lateral extensometer, is shown in Figures 10 to 12.

Figure 14, which shows the maximum stress contours determined by two different methods, illustrates the similarity of results. Even though the shape of the contour sets are not exactly similar, the general location of maximum stresses is certainly conclusive. Furthermore, Sandel loaded several plastic models to failure and in each case the plane of failure was almost identical with that which occurred in the larger masonite model (Figure 13).

Comparison With Analytical Results

In a careful analytical study of stresses in a gusset plate the following items are usually investigated: flexure stress, direct stress, and shearing stress on the "critical section"; tearing out of the tension diagonal at the end of the member; and localized crushing of the plate near the end of the compression diagonal.

The "critical section" referred to above should be taken parallel to one of the intersecting members³ and should be so located that

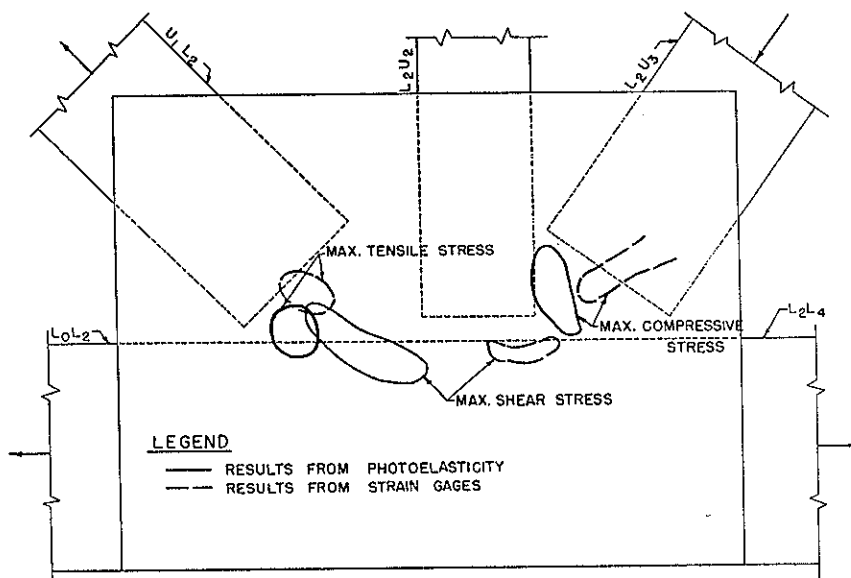
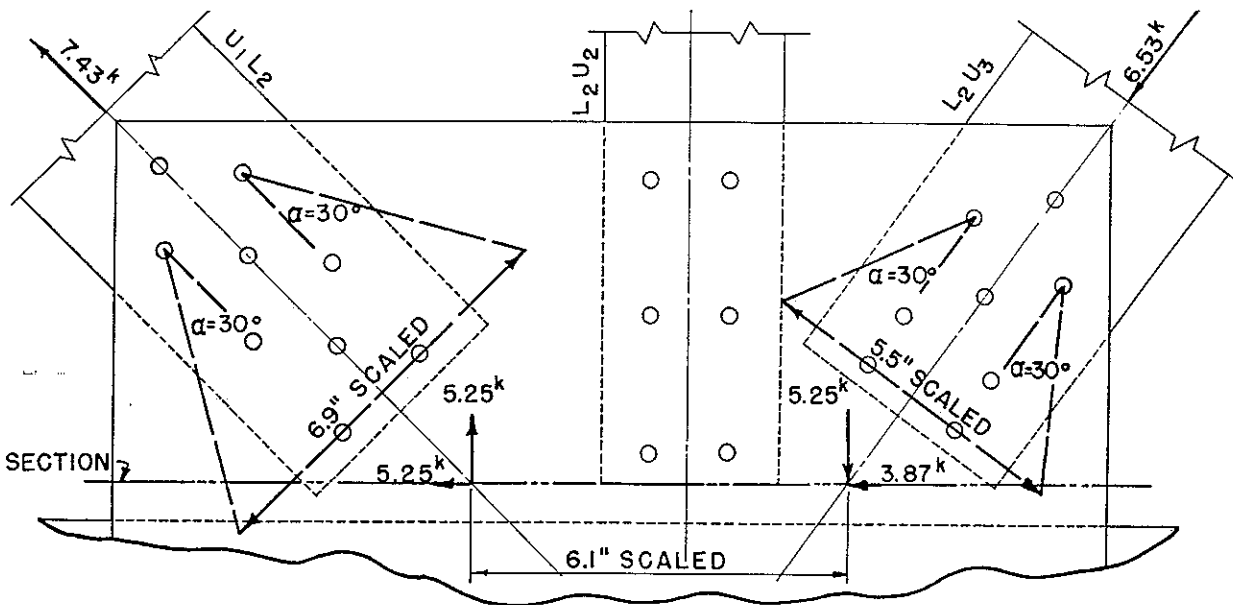


Figure 14. Comparison of Maximum Stresses from Different Test Methods.

the stresses thereon are a maximum. Careful study of forces causing stresses in the model resulted in the conclusion that the "critical section" in this case is one that is parallel to the bottom chord and through the ends of each diagonal as shown in Figure 1. On this section the vertical component of each diagonal contributes to the bending, and the horizontal components are a maximum and act in the same direction to increase the shearing stress. On the assumption that the usual beam formulas apply, the bending and shearing stresses may be calculated.

Figure 15 shows the part of the plate above the "critical section", the components of the forces, and the calculation of the stresses. The bending stresses are plotted in Figure 18 and the shearing stresses in Figure 19.

To obtain a comparison with these analytical results, similar stresses were determined from the experimental data. From Figures 6, 7, 8, and 9, values of p , q , τ , and θ were obtained at various points along the "critical section." Curves showing the distribution of these four items along the section were plotted in Figures 16 and 17. Using data taken from these curves, normal and shearing stresses then were calculated (at each one-half-inch interval along the "critical section") by means of the following established relationships:



$$\text{BENDING STRESSES } \sigma_{Y(\max)} = \frac{M c}{I} = \frac{(5.25)(6.1)(6)}{(0.25)(16.6)^2} = 2.78 \text{ k/sq.} = 2,780 \text{ P.S.I.}$$

$$\text{SHEARING STRESSES } \tau_{XY(\max)} = \frac{3}{2} \frac{V}{A} = \frac{(3)(9.12)}{(2)(0.25)(16.6)} = 3.30 \text{ k/sq.} = 3,300 \text{ P.S.I.}$$

$$\text{DIRECT TENSION } \sigma = \frac{P}{A} = \frac{7.43}{(0.25)(6.9)} = 4.31 \text{ k/sq.} = 4,310 \text{ P.S.I.}$$

$$\text{DIRECT COMPRESSION } \sigma = \frac{P}{A} = \frac{6.53}{(0.25)(5.5)} = 4.76 \text{ k/sq.} = 4,760 \text{ P.S.I.}$$

Figure 15. Calculation of Stresses in the Model Based on Flexure Theory.

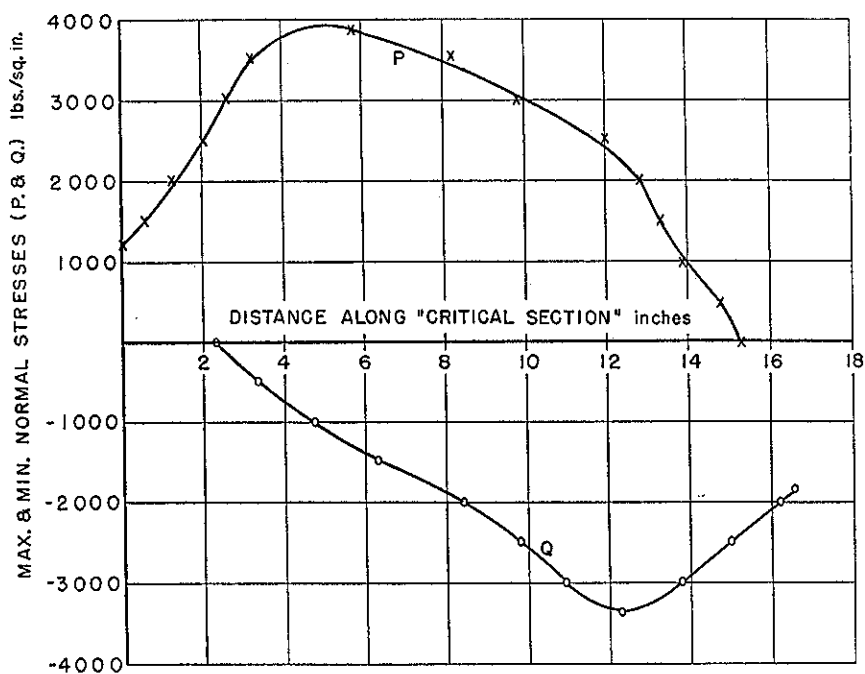


Figure 16. Distribution of p and q on the "Critical Section." Experimental Results.

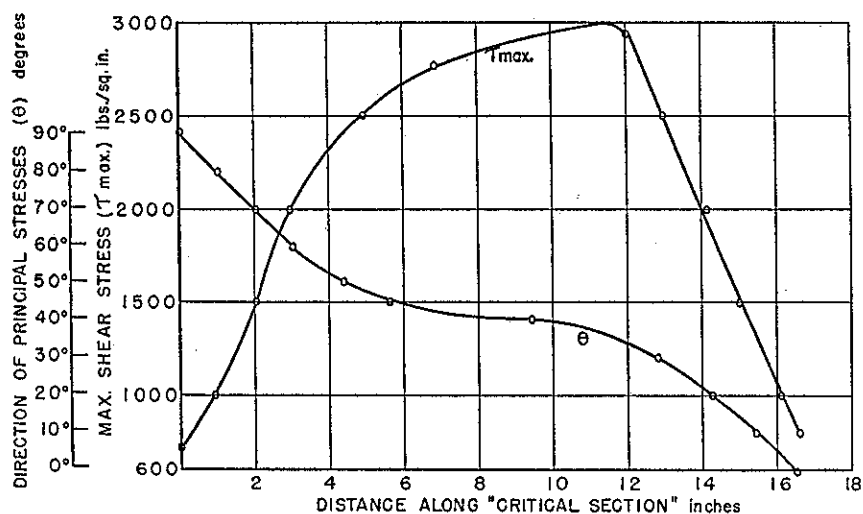


Figure 17. Distribution of θ and τ on the "Critical Section." Experimental Results.

$$\sigma_y = \frac{p+q}{2} - \frac{p-q}{2} \cos 2\theta$$

$$\tau_{xy} = \frac{p-q}{2} \sin 2\theta$$

The values obtained are given in Table 5 and the results are plotted in Figures 18 and 19.

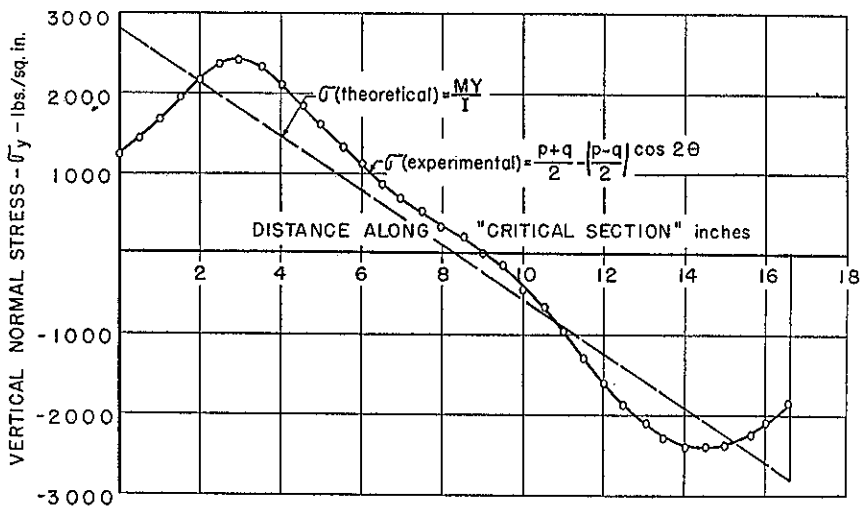


Figure 18. Distribution of Vertical Normal Stress on the "Critical Section."

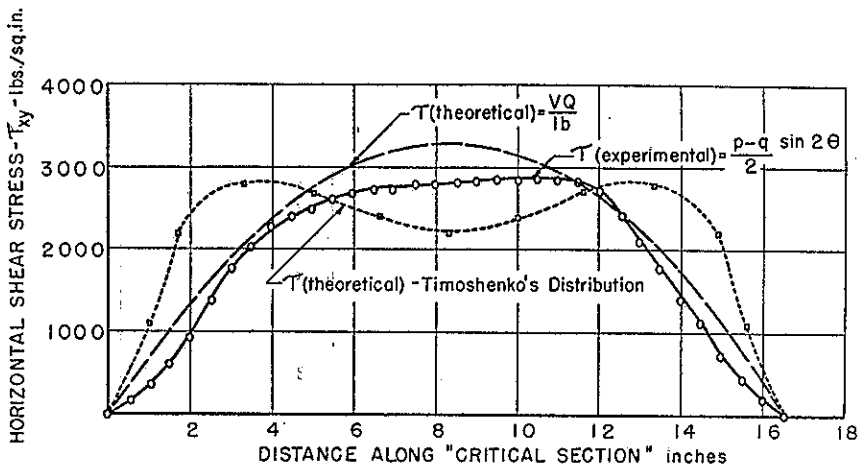


Figure 19. Distribution of Horizontal Shearing Stress on the "Critical Section."

TABLE 5
Stresses on the "Critical Section"

Distance from Left Edge of Plate (inches)	$\frac{p-q}{2}$ (lb per sq in)	$\frac{p+q}{2}$	$\sin 2 \theta$	$\cos 2 \theta$	T_{xy}^a (lb per sq in)	σ_y^b (lb per sq in)
0	625	625	0.000	-1.000	0	1,250
$\frac{1}{2}$	735	735	0.174	-0.985	130	1,460
1	875	875	0.341	-0.940	300	1,700
$1\frac{1}{2}$	1,050	1,050	0.500	-0.866	530	1,960
2	1,235	1,235	0.641	-0.766	790	2,180
$2\frac{1}{2}$	1,520	1,400	0.765	-0.643	1,170	2,380
3	1,850	1,485	0.866	-0.500	1,610	2,410
$3\frac{1}{2}$	2,080	1,530	0.926	-0.374	1,930	2,310
4	2,260	1,525	0.960	-0.242	2,170	2,070
$4\frac{1}{2}$	2,400	1,500	0.990	-0.139	2,380	1,840
5	2,510	1,455	0.996	-0.069	2,500	1,630
$5\frac{1}{2}$	2,580	1,350	1.000	0.000	2,580	1,350
6	2,620	1,230	0.997	0.052	2,620	1,100
$6\frac{1}{2}$	2,640	1,125	0.996	0.087	2,630	900
7	2,650	1,025	0.990	0.122	2,630	700
$7\frac{1}{2}$	2,670	900	0.989	0.139	2,640	530
8	2,680	795	0.984	0.174	2,640	330
$8\frac{1}{2}$	2,700	675	0.984	0.174	2,660	210
9	2,720	535	0.980	0.191	2,670	20
$9\frac{1}{2}$	2,740	385	0.978	0.208	2,680	-180
10	2,780	235	0.970	0.242	2,700	-440
$10\frac{1}{2}$	2,820	75	0.965	0.259	2,720	-660
11	2,890	-140	0.956	0.293	2,770	-990
$11\frac{1}{2}$	2,910	-300	0.940	0.342	2,740	-1,300
12	2,900	-440	0.926	0.391	2,690	-1,580
$12\frac{1}{2}$	2,830	-615	0.891	0.454	2,530	-1,900
13	2,580	-725	0.848	0.530	2,200	-2,090
$13\frac{1}{2}$	2,250	-895	0.788	0.616	1,780	-2,290
14	1,960	-965	0.695	0.720	1,370	-2,380
$14\frac{1}{2}$	1,680	-1,040	0.630	0.776	1,060	-2,350
15	1,400	-1,125	0.485	0.875	680	-2,350
$15\frac{1}{2}$	1,175	-1,175	0.341	0.940	400	-2,280
16	1,075	-1,075	0.174	0.985	190	-2,140
16.6	925	-925	0.000	1.000	0	-1,850

$$^a T_{xy} = \frac{p-q}{2} \sin 2 \theta$$

$$^b \sigma_y = \frac{p+q}{2} - \frac{p-q}{2} \cos 2 \theta; \text{ tension is plus and compression is minus.}$$

As mentioned above, gusset plates are sometimes analyzed for tearing out and crushing at the ends of the diagonals. As illustrated in Figure 15, this may be accomplished by assuming that the forces in the diagonals are distributed uniformly over an area of plate material obtained by multiplying the thickness of plates by an arbitrary length perpendicular to the diagonals. This length is obtained by extending lines (originating in the outer rivets of

the first row), making an angle "oc" with the direction of the member, until they intersect a line perpendicular to the axis of the member and passing through the bottom row of rivets. The intercept can then be scaled. An "oc" angle of 30 degrees is commonly used.

This construction and the resulting stresses are shown in Figure 15. The stresses may be compared with those plotted in Figures 6 and 7.

Statical Checks

An additional check on the experimental results may be obtained by applying the equations of statics, namely:

$$\Sigma H = 0$$

$$\Sigma V = 0$$

$$\Sigma M = 0$$

These equations were applied to check stresses on the "critical section."

The total horizontal shear on the section as shown by the experimental data may be obtained by multiplying the average shearing stress computed from the tabulated value of τ_{xy} by the area of the plates on the critical section. This gives 7.75^k. The actual horizontal shear is 9.12^k, so the experimental value is about 15 per cent too small.

Figure 20 was plotted so that the other equations of statics might be applied. As the plotted values are vertical normal stresses, the summation of all forces should equal zero for equilibrium of vertical forces. As shown in the figure, areas have been converted to forces and the results are:

$$V = 3.23^k - 3.08^k = 0.15 \text{ kips}$$

$$\text{Per cent error (based on average)} = 4.7$$

Using the same figure, a comparison of moments may be made.

The experimental results indicate the resisting moment to be:

$$M = 3.23 \times 5.48 + 3.08 \times 4.72 = 32.2 \text{ inch-kips}$$

From Figure 15 the theoretical moment is seen to be:

$$M = 5.25 \times 6.1 = 32.0 \text{ inch-kips}$$

$$\text{Per cent error} = 0.6$$

It is interesting that the same experimental data (p, q, and θ) enter into the calculation of all stresses checked above. However,

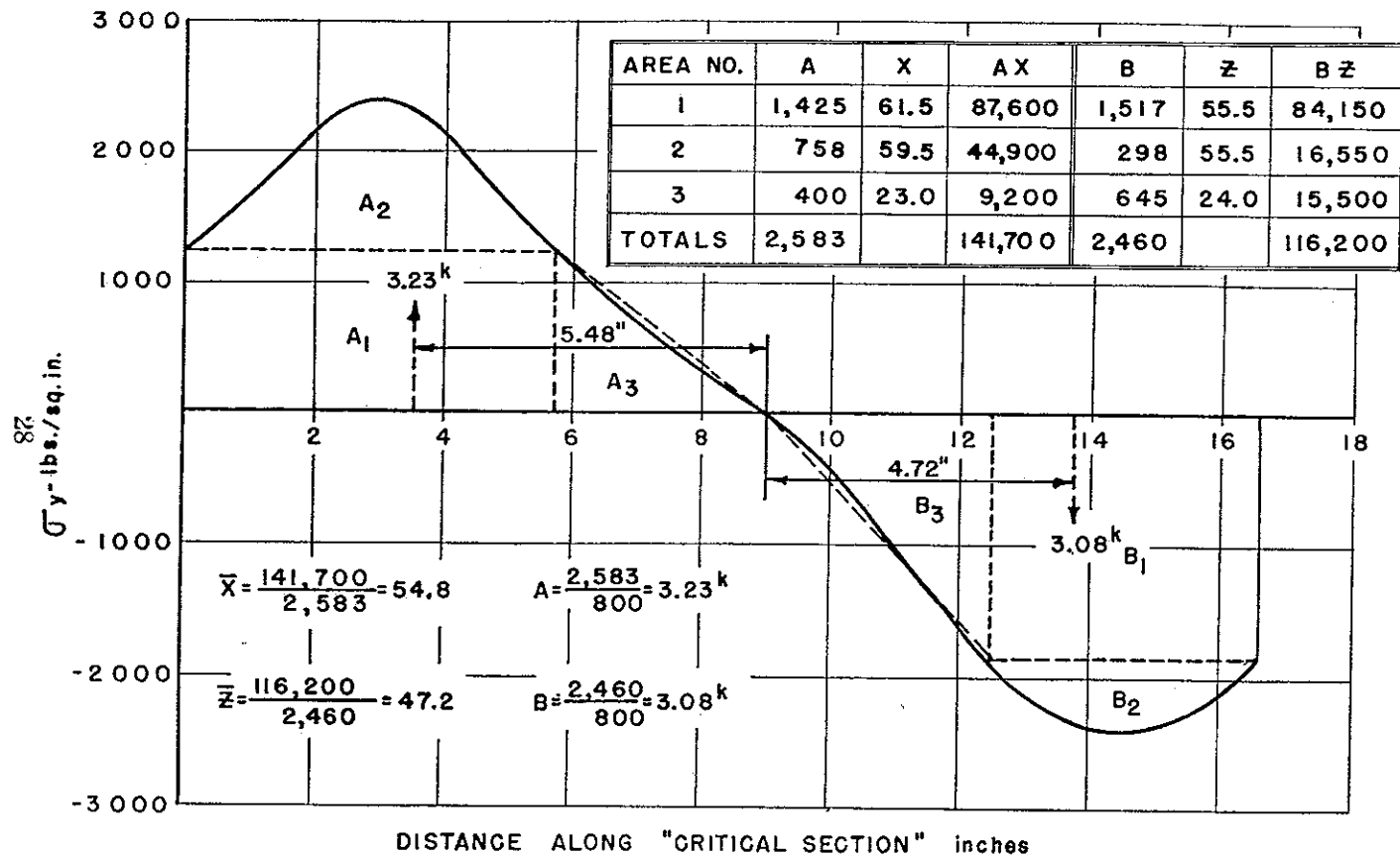


Figure 20. Determination of the Resisting Moment on the "Critical Section."

considerable variation in the percentage of errors is evident, the greatest discrepancy being in the horizontal shear. Therefore, it is not believed that the experimental data are in error by as much as 15 per cent; one-half of this might be a more reasonable figure. In view of the many possible sources of error, including plotting and scaling from contour lines, the discrepancies shown are thought to be not excessive.

In general, it can be stated that the experimental results of this investigation have been verified by cross-checks between the different runs, by checks between different test methods, and by statics.

CONCLUSIONS

In drawing conclusions from the results of the experiments performed in this study it should be remembered that they may be applied directly only to joints of a similar type—namely, joints of a Warren truss in which the chord direction does not change at the joint and in which the chord is not spliced in the joint. However, a great many actual joints are of this type.

A study of the results obtained seemed to justify the following conclusions regarding gusset plates of the type investigated:

1. The general distribution of stresses is as shown in Figures 5 to 8. A study of the stress trajectories alone gives an excellent visualization of the stress distribution. This distribution appears to be that which would naturally be expected when the nature of the applied forces is studied carefully.
2. The location of the maximum tensile stress is at the end of the tension diagonal and the maximum compressive stress is at the end of the compression diagonal. The maximum shearing stress is on a plane slightly below the ends of the diagonals, the maximum value occurring approximately half way between the ends of the diagonals.
3. The assumption that the direct, bending and shearing stresses on a plane through the ends of the diagonals are distributed according to the beam formulas, $\sigma = P/A \pm Mc/I$ and $\tau = VQ/Ib$, is fallacious. This will probably come as a surprise to few engineers. T. C. Shedd⁹ has said in regard to these formulas:

... the student should be warned against placing too much reliance on these estimated stresses in the gussets. They are necessarily based on the assumption that the beam formula holds: the accuracy of the beam formula when applied to short deep beams, where highly localized effects occur, is quite uncertain."

An examination of the joint makes it quite evident that the applied forces are too near the stressed section for St. Venant's principle to apply: consequently, it is obvious that the beam formula will give misleading results. Regarding flexure stresses, the greatest error is in the edge stresses; the beam formula shows these to be maximum, whereas the experimental investigation shows clearly that this is not the case. For the shearing stresses, the general distribution is in greater error than the maximum value (which is about correct when calculated by the usual method). Figure 19 clearly shows the difference. The third curve plotted is one suggested by Timoshenko¹⁰ for stresses on a section near the applied forces. It is seen that the experimental distribution lies between this and the usual parabolic curve.

4. If the beam formulas are not to be used to calculate stresses, how may they be determined? Test results indicate that the most rational method of determining maximum normal stresses is by the method illustrated in Figure 15, in which it is assumed that the stresses are uniformly distributed over an area at the end of each diagonal. At first it might appear that using a "spread out" angle " α " of 30 degrees is conservative, since the calculated stresses are slightly larger than the measured stresses. However, the stress concentrations at the rivet holes were not measured (the gages used for this purpose apparently were not close enough to the holes to show the effect of stress concentration) and it is well known that maximum stresses at the edges of the holes are greater than those in the surrounding area of the plate. Therefore, it is believed that an " α " angle of 30 degrees is reasonable and may well be used in the practical determination of the maximum stress.

It should be noted that for this joint the tensile stresses are more apt to cause trouble than the compressive stresses.

Apparently the maximum shear stresses can be calculated with reasonable accuracy by the usual method, the maximum value being one and one-half times the average shearing stress.

5. The fact that the compressive stresses at the edges of gusset plates of this type are much lower than commonly assumed indicates that serious consideration may well be given to the omission of stiffener angles at such edges except in unusual cases. General practice in the past has apparently been unnecessarily conservative in this respect.

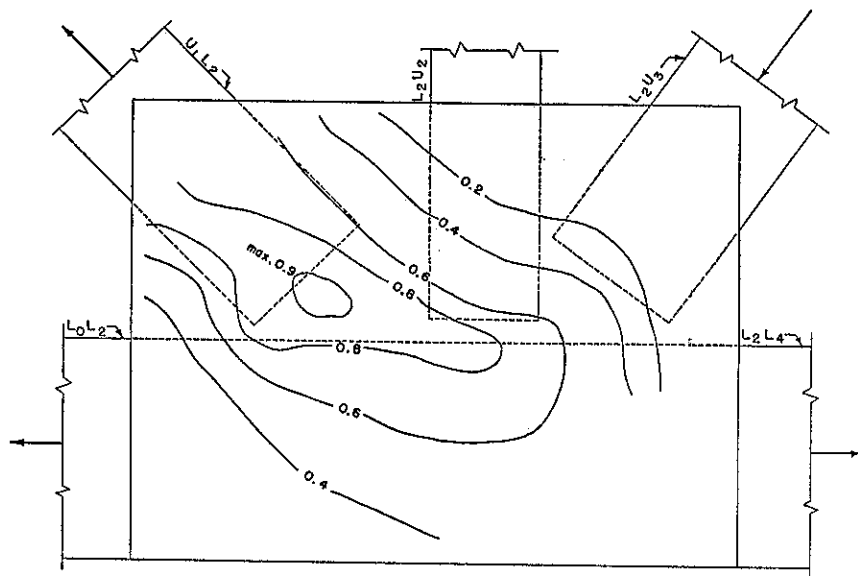


Figure 21. Contours of p , Expressed as Ratios of P/A .

To aid designers in the analytical study of stresses in similar plates, Figures 21 to 23 have been prepared. The contour values are ratios of the P/A stress to the actual stress as determined experimentally. P/A for the tension set of contours was determined by dividing the force in the tension diagonal by the "spread out" area at the end of the member determined as previously explained. A similar procedure was used for the compressive set of contours. For the shear values, " P " was taken as the shear force on the plane and " A " as the cross-sectional area of the plates.

To determine the stresses in any particular plate, these ratios may be multiplied by the particular value of P/A for the case

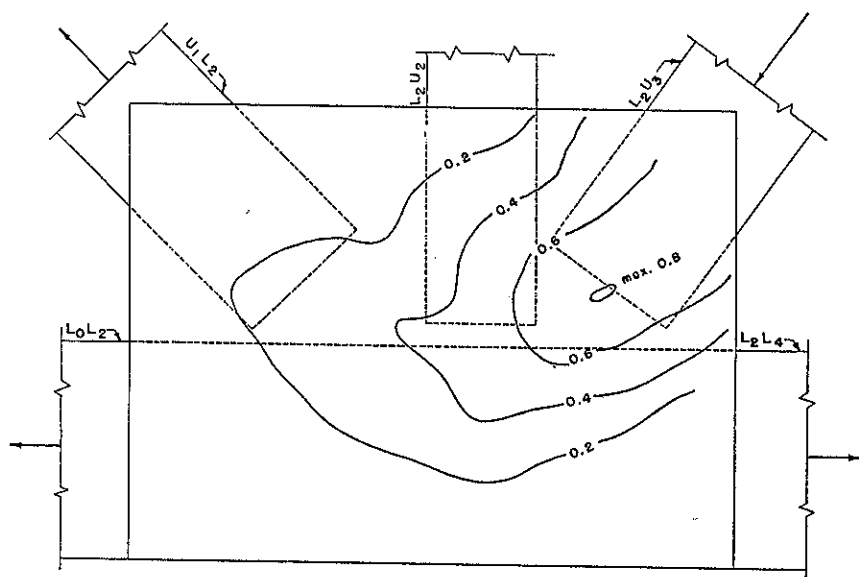


Figure 22. Contours of q Expressed as Ratios of P/A .

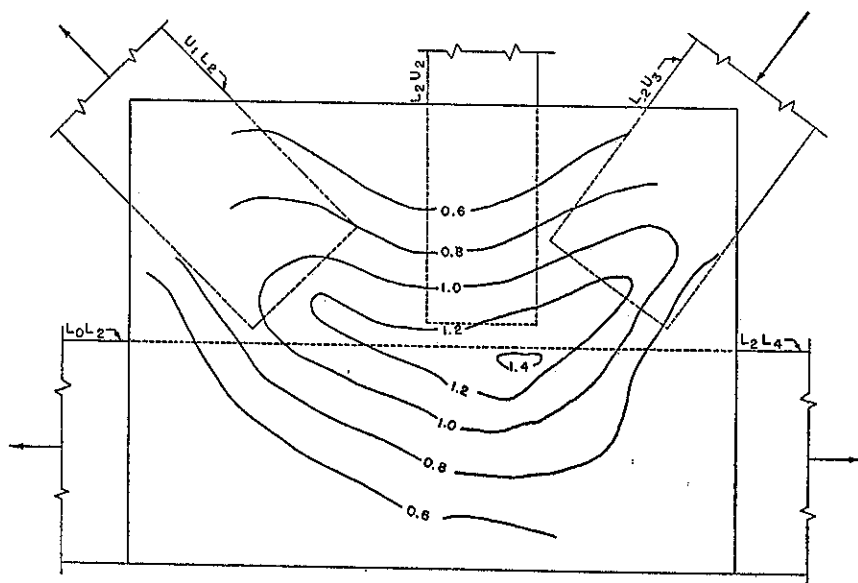


Figure 23. Contours of τ Expressed as Ratios of P/A .

under consideration. In all probability, the maximum value will usually be the only value checked. This provides a simple and reliable method of designing or checking gusset plates of the type studied herein.

Undoubtedly, further testing of gusset plates is necessary before generalized conclusions may be drawn. This would involve testing joints of other types, sizes and shapes, and studying the effects of secondary stresses in the members in addition to the direct stresses, so that all gusset plates may be proportioned simply, safely and economically. However, the results of the tests made on the gusset plates of this research are quite significant, and they may be applied with considerable confidence to the analysis of stresses in gussets similar to the one tested.

REFERENCES

1. Perna, F. J., "Photoelastic Stress Analysis With Special Reference to Stresses in Gusset Plates," Master's Thesis, Univ. of Tenn., 1941.
2. Sandel, J. A., "Photoelastic Analysis of Gusset Plates," Master's Thesis, Univ. of Tenn., 1950.
3. Whitmore, R. E., "Experimental Investigation of Stresses in Gusset Plates," Master's Thesis, Univ. of Tenn., 1950.
4. Rust, T. H., "Steel Gusset-Plates." Proceedings of the Thirteenth Semi-Annual Eastern Photoelastic Conference (Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts, 1941), p. 55.
5. Alcoa Structural Handbook, Aluminum Co. of America (Pittsburgh, 1945).
6. Tatnall, F. G., "Concerning the Subject of Testing," SR-4 News Letter, Vol. II, No. 5, 1946.
7. Lee, G. H., An Introduction to Experimental Stress Analysis (New York: John Wiley and Sons, 1950), p. 54.
8. Waddell, J. A. L., Bridge Engineering (New York: John Wiley and Sons, 1916), I: 521-524.
9. Shedd, T. C., Structural Design in Steel (New York: John Wiley and Sons, 1934), pp. 349-351.
10. Timoshenko, S., Theory of Elasticity (New York: McGraw-Hill Book Co., 1934), p. 49.